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P. McEachern

W. J. Gage & Co.'s Educational Series.

HINTS AND ANSWERS

TO

EXAMINATION PAPERS

SUITABLE FOR

INTERMEDIATE EXAMINATION.

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PREFACE.

It was the original intention of the Compilers of the *Examination Papers suitable for the Intermediate Examination* to publish Hints and Answers to all the questions that presented any difficulty, but it has been suggested to them that this course was not necessary when the Answers could easily be found by reference to a good text-book—a suggestion which they adopted. In consequence, answers were given to some of the early papers in English Literature, Grammar and Geography, but not to later ones, and for the same reason no answers appear to the Euclid, Book-keeping and History papers.

We consider it advisable, however, to republish all the papers as they appeared in *Gage's School Examiner and Student's Assistant* with the hope that the enterprise will prove of practical benefit to those for whom the exercises are intended.

April, 1882.

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HINTS AND ANSWERS TO

INTERMEDIATE EXAMINATION PAPERS.

———:(o):———

Arithmetic.—No. 1.

1. Amount of debts $=2\times\$21,735=\$43,470$;

$$\therefore \frac{105}{100} \text{ of } \frac{103\frac{1}{2}}{100} \text{ of capital } =\$43,470;$$

$$\therefore \text{capital}=\tfrac{100}{105} \text{ of } \tfrac{200}{207} \text{ of } \$43,470,$$
$$=\$40,000.$$

Hence A's share $=\frac{3}{5}$ of $\$40,000=\$24,000$;
and B's " $=\frac{2}{5}$ of $\$40,000=\$16,000$.

2. No change.

3. The two nails will touch the rails together after going 803 feet, the L. C. M. of the number of feet in the circumference of each wheel;

$$\therefore \text{ time required}=\frac{803\times 60}{6\frac{1}{4}\times 5280} \text{ minutes.}$$
$$=1\tfrac{23}{50} \text{ minutes.}$$

4 $$\frac{\sqrt[3]{1.92}-24\sqrt[3]{.00004562625}}{\sqrt[3]{.81}+\sqrt[3]{.03}}=\frac{\sqrt[3]{.03}(\sqrt[3]{64}-24\sqrt[3]{.001520875})}{\sqrt[3]{.03}(\sqrt[3]{27}+\sqrt[3]{1})}$$
$$=\frac{4-24\times .115}{3+1}$$
$$=1-6\times .115$$

5. Area of end $=(3\frac{1}{7}\times 9\times 9)$ square inches.
Cubic contents of solid cylinder $=(38\times 3\frac{1}{7}\times 9\times 9)$ cubic inches.
$=9673\frac{5}{7}$ cubic inches.
Area of inside of end $=(3\frac{1}{7}\times 8\times 8)$ square inches.
Cubic contents of hollow $=(36\times 3\frac{1}{7}\times 8\times 8)$ cubic inches.
$=7241\frac{1}{6}$ cubic inches.
Quantity of iron $=2432\frac{4}{7}$ cubic inches.
Weight of iron $=2432\frac{4}{7}\times 7\frac{7}{10}\times\frac{1000}{1728}$ ounces.

6. Let r = the interest of \$1 for the fourth of a year,
then $(1+r)^4=1.08$;
$\therefore 1+r=\sqrt[4]{1.09}=1.019\ldots$;
$\therefore r=.019\ldots$ and the rate per cent $=1.9\ldots$

7. $5\frac{2}{3}\%$.

8. \$5.03 (nearly).

9. The sum of the velocities $=\dfrac{120+150}{3}$ feet per second
$=90$ feet.
The difference " " $=\dfrac{120+150}{10}$ " "
$=27$ feet.
The velocity of faster $=\dfrac{90+27}{2}$ " "
" slower $=\dfrac{90-27}{2}$ " "

The rate per hour can easily be obtained from these.
Ans. $39\frac{39}{44}$ miles and $21\frac{21}{44}$ miles.

No. 2.

1. In Canada the French system of notation prevails. In it there are 3 figures in a period; in the English system 6 figures make a period.

1,000000,000000 is an English billion. The two systems correspond till the place of hundreds of millions is reached; in English notation the next place is called thousands of mil-

lions ; the next tens of thousands of millions, etc. An English billisn is thus equal to 1000 of our billions.

2. $\$36.66\frac{2}{3}$.

3. Cost of one-half wheat per bushel $=\frac{100}{120}$ of $1.20=$1.

$$\text{Cost of other half per bushel}=\tfrac{100}{80}\text{ of }\$1.20=\$1.50$$

$$\therefore \text{cost of two bushels}=\$2.50\ ;$$

$$\therefore \text{loss on }\$2.50=\$.10\ ;$$

$$\therefore \text{loss on }\$100=\frac{100\times\$.10}{2.50}=\$4.$$

4. Since he gave ($\frac{1}{2}$ of property—\$12000) to eldest son, first remainder=($\frac{1}{2}$ of property+\$12000).

Second " $=\frac{2}{3}$ of ($\frac{1}{2}$ of property+\$12000)—\$6000.

Third " $=\frac{1}{3}$ of $\{\frac{2}{3}$ of ($\frac{1}{2}$ of prop. + \$12000)–\$6000$\}$ + \$2000

$=\frac{1}{9}$ of property + $\$2666\frac{2}{3}$—\$2000+\$2000.

$\therefore \frac{1}{9}$ of property+$\$2666\frac{2}{3}$=\$12000.

From this his property is found to be \$84000. Share of 1st, \$30000 ; 2nd, \$24000; 3rd, \$18000.

5. Since the areas of similar surfaces are to each other as the squares of their like dimensions, we have

$$48^2 : (\text{length of 2nd})^2 :: \$500 : \$320\ ;$$

$$\therefore \text{length of 2nd}=\sqrt{\frac{48^2\times 320}{500}}$$

$$=\frac{48\times 4}{5}$$

$$=38\tfrac{2}{5}.$$

6. Interest on \$3955 for 6 months=$\$138.42\frac{1}{2}$.

Amount of stock sold $=\$\dfrac{3955\times 100}{98\frac{7}{8}}$

$=\$4000.$

Money necessary to buy \$4000 $=\$\dfrac{4000\times 97\frac{1}{2}}{100}$

$=\$3900.$

Sum to be paid $=\$3900+\$138.42\frac{1}{2}$.

$=\$4038.42\frac{1}{2}$.

7. The interest on \$760 for 93 days, at 7%, is \$13.55 ; the amount of the note is \$773.55 ; the interest on this sum for 65 days is \$773.55—\$759.78. From this the rate per cent. per annum can easily be found. *Ans.* 10%.

8. $18\frac{6}{13}$ hours.

9. Since the diameter of the sphere is the diagonal of the cube, therefore the edge of cube

$$=\frac{12}{\sqrt{3}}\text{ inches.}$$
$$=4\sqrt{3}\text{ ''}$$
$$=6.928\text{ ''}$$

Solidity of sphere $$=\frac{3.1416\times 12\times 12\times 12}{6}$$
$$=904.7808$$

Solidity of cube $$=(4\sqrt{3})^3$$
$$=332.544$$

$\therefore$ weight of cuttings $=\frac{657}{1000}$ of 572.2368 of $\frac{1000}{1728}$ oz.

10. *Ans.* $1114\frac{2}{3}$ feet per second.

No. 3.

1. Seven, and ten million twenty thousand and thirty one ten-billionths.

17.0000001008006.

2. Resolve 171685800 into its prime factors : one of these will be found to be 29. As 29 is not a factor of either of the first two numbers it must be one of the third.

3. \$4500.

4. A's investment=\$5600 for 1 mo.
B's " =\$7400 " "
C's " =\$14600 " "
C's gain =\$1710—(\$430 + \$550)
=\$730.

Capital required to gain $730=$14600 for 1 mo.

" " " $\$430=\$\frac{430 \times 14600}{730}$ "

$=\$8600$;

∴ sum put in by A $=\$\frac{8600-5600}{6}$

$=\$500$.

Similarly B's sum is found to be $600.

5. $2\frac{1}{4}$ oz.

5 If he has 16 parts of the good article, he mixes four parts of the inferior with them and sells the whole for 22 parts. But the four parts of the inferior cost the same as $3\frac{1}{5}$ parts of the better, hence he sold $19\frac{1}{5}$ parts for 22 parts. From this his gain is found to be $14\frac{7}{12}\%$. To gain 20%, what he sells for 22 must cost $18\frac{1}{3}$. Then cost of the ingredients being 20 and 16 what portion of each must be taken to form a mixture worth $18\frac{1}{3}$. This is easily found to be 7 and 5.

7. 10 in.

8. The interest for a certain time at $(10\frac{1}{2}-8)\%$ is $1012.50-$950 or $62.50.

Interest at $2\frac{1}{2}\%=\$62.50$

∴ " " $8\%=\$\frac{8 \times 62.50}{2\frac{1}{2}}$

$=\$200$.

Hence the principal is $950-$200 or $750.

The time is easily found to be $3\frac{1}{3}$ years.

9. Sum left after gaining $\$14=\frac{3}{4}$ of money + $14.

" " " " $\$8=\frac{4}{5}(\frac{3}{4}$ of money$+\$14)+\$8$

$=\frac{3}{5}$ of money$+\$19\frac{1}{5}$;

∴ $\frac{2}{5}$ of money $=\$19\frac{1}{5}$

Hence his money $=\$48$.

∴ $\frac{2}{5}$ of money $=\frac{3}{5}$ of money $+\$19\frac{1}{5}$

$=\$19\frac{1}{5}$.

10. Take the square root of the number, then extract the cube root of the result; or take the cube root of the number and extract the square root of the result.

Ans. 101.

No. 4.

1. $11\frac{191}{819}$.

2. Since 5 men, 4 boys, and 3 girls clear $\frac{1}{9}$ of field in 1 day,

$\therefore$ 10 men, 8 boys. and 6 girls 〃 $\frac{2}{9}$ 〃 〃

But 3 men, 8 boys, and 6 girls 〃 $\frac{1}{8}$ 〃 〃

$\therefore$ 7 men clear $(\frac{2}{9}-\frac{1}{8})$ of field in 1 day

And 1 man clears $(\frac{7}{72}\div 7$ or $\frac{1}{72})$ of field in 1 day.

Again, since 15 men, 12 boys, and 9 girls clear $\frac{1}{3}$ of field in 1 day

And 4 men, 10 boys, and 9 girls $\frac{1}{6}$ 〃 〃

$\therefore$ 11 men, and 2 boys clear $(\frac{1}{3}-\frac{1}{6}$ or $\frac{1}{6})$ 〃

But 11 men clear $\frac{11}{72}$ of field in 1 day

$\therefore$ 2 boys clear $(\frac{1}{6}-\frac{11}{72}$ or $\frac{1}{72})$ of field in 1 day

And 1 boy clears $\frac{1}{144}$ of field in 1 day.

In a similar manner it may be found that 1 girl clears $\frac{1}{216}$ of field in 1 day.

Hence 3 men, 5 boys, and 4 girls do $(\frac{3}{72}+\frac{5}{144}+\frac{4}{216})$ of work in 1 day, &c.

Ans. $10\frac{22}{41}$.

3. The machine cost $40 before the duty was paid.

4. Since $\frac{98}{100}$ of child's share $=2$ ($\frac{96}{100}$ of brother's share)

$\therefore$ 4 children's shares $=8\times\frac{100}{98}\times\frac{96}{100}$ of brother's share

$=7\frac{41}{49}$ brothers' shares.

Ans. $4800 = a child's share and $2450 = a brother's share.

5. Cost of sugar $=2000\times 8$ cents $+\$13$

$=\$173$.

Cost at 7cts. $=\$140$

The difference between the actual cost and the cost at 7 cents, viz. $33, arises from having bought a part at 3 cts. per lb. more than 7 cts.

Number of lbs. at 10 cts $= \dfrac{\$33}{3\text{cts}} =$ 1100 lbs.

Number at 7cts $=$ 900 lbs.

7. Amount of note at the end of year $= 400 \times \$1.10$
$= \$440$

Interest on \$390 for 9 mos. $= \$440 - \$390$
$= \$50$

From this the rate per annum can be found.
Ans. $17\frac{11}{117}\%$.

8. He buys \$6000 stock. He receives \$300 for his dividend.
Sum received $= \$5970 + \$300 = \$6270$.
Sum expended $= \$6135 + \frac{7}{100}$ of $\$6135 = \$6564.45$.
Loss $= 6564.45 - \$6270 = \$294.45$.

9. $235892\frac{28}{241}$ fr.

10. *A* overtakes *B* $61\frac{7}{13}$ yds. from the corner where *B* started : 3 times.

No. 5.

1. ·3 ; ·1.

2. Suppose the vulgar fraction to be in its lowest terms. The division is carried on by successively multiplying the numerator by 10. The factors of 10 are 2 and 5. Hence, if the denominator contains no factors but 2 or 5, or powers of these, the decimal will terminate; if it has other factors than these, it will be a repeating decimal.

3. 12 bales and 21 casks fill $\frac{3}{5}$ of cave,
∴ 4 " 7 " $\frac{1}{5}$ "
and 20 " 35 " the cave.
But 18 " 40 " "
∴ 2 bales occupy the same space as 5 casks,
and 1 bale occupies " " $2\frac{1}{2}$ "
∴ 18 bales occupy " " 45 "
Hence it would hold $(45+40)$ casks,
or $(18+16)$ bales.

4
$$\tfrac{1}{5} \times \tfrac{105}{100} = \tfrac{21}{100} \text{ cost price,}$$
$$\tfrac{1}{4} \times \tfrac{107}{100} = \tfrac{107}{400} \quad \text{"}$$
$$\tfrac{11}{20} \times \tfrac{112}{100} = \tfrac{1232}{2000} \quad \text{"}$$

Total sum realized $= (\tfrac{21}{100} + \tfrac{107}{400} + \tfrac{1232}{2000})$ of cost.

Sum realized at 9% $= \tfrac{109}{100}$ of cost,

$\therefore \quad (\tfrac{2187}{2000} - \tfrac{109}{100})$ of cost $= \$24.50$

Ans. $7000.

5. Face of 1st note $= \$(\tfrac{100}{98}$ of $375) = \$382\tfrac{32}{49}$

Present value of 2nd note $= \$(382\tfrac{32}{49} + 1.75)$

Face of 2nd note $= \$(\tfrac{100}{98}$ of $384\tfrac{79}{196})$

$= \$392.25$ cts. nearly.

6. Since $\{\tfrac{6}{106} - (\tfrac{4}{108} + \tfrac{2}{104})\}$ of sum $= \$50$,

$\therefore$ sum $= \$148824$.

7. Sum realized $=$ £30400000.

Total interest paid $=$ £1830000,

$$\therefore \quad \text{average rate} = £\frac{100 \times 1830000}{30400000}$$
$$= 6\tfrac{3}{152}.$$

Interest paid on each £ of 1st $=$ £$\tfrac{5}{80}$; of 2nd, £$\tfrac{4}{70}$; of 3rd, £$\tfrac{7}{120}$.

$\therefore$ the lowest rate is paid on the second.

9. Since 216 (cost of 1 oz. of gold + cost of 1 oz. of silver)
$=$ £637 7s. $+$ £209 1s.

$$\therefore \text{cost of 1 oz. of gld} + \text{cost of 1 oz. of silver} = \frac{£896\ 8s.}{216}$$
$$= £4\ 3s.$$

$\therefore$ cost of 1 oz. of silver $=$ £4 3s. $-$ £3 17s 10½d.
$=$ 5s. 1½d.

By a process similar to that employed in the solution of problem 5 of the April number, it is found that there are 160 oz. of gold and 56 oz. of silver, hence the proportions are as 20 to 7.

10. $15\tfrac{5}{8}$ per cent. of wire ; $84\tfrac{3}{8}$ per cent. of hemp.

No. 6.

1. See Kirkland and Scott's Elementary Arithmetic, Art. 141. Ans. 27.

2. L. C. M. of 2, 3, 4, 5, 6=60; 60 + 1=61.

This number is not divisible by 7; we must, hence, find the least multiple of 60, which increased by 1 is exactly divisible by 7. Ans. 301 peaches.

3. Each time he draws off $\frac{3}{10}$ of the wine then in the cask. Part remaining $= \frac{7}{10}$ of $\frac{7}{10}$ of $\frac{7}{10}$ of $\frac{7}{10}$ of 20 gal. = 4.802 gal., etc.

4. If he owns \$100 stock his tax is 4 cents and his income is reduced to \$3.96, and when the tax is doubled, to \$3.92. But when \$100 stock yields \$3.96 net-income, he is making 4 per cent. on his investment; what per cent. is he making when \$100 stock yields him \$3.92 net-income?

$$\text{Per cent.} = \frac{392 \times 4}{396} = 3\tfrac{95}{99}.$$

5. By Alligation it is readily shown that 11 men, 8 women and 8 boys would receive on an average \$1.10 per day. Hence number employed = 22 men + 16 women + 16 boys.

6. 1s.; $6\frac{2}{3}$ %.

7. 7 min. 14 $\frac{766}{1201}$ sec. past 11.

8. 20 per cent.

9. Water displaced by 1 oz. silver $= \frac{2304}{15000}$ c. in.
" " " " gold $= \frac{1296}{15000}$ c. in.
Excess of water displaced by 1 oz. silver $= \frac{1008}{15000}$ c. in.
Water displaced by alloy = 13.824 c. in.
" " " gold = 12.96 c. in.
Excess of water displaced by alloy = .864 c. in.
Hence, number of ounces of silver $= .864 \div \frac{1008}{15000}$.
$= 12\frac{6}{7}$.
Number of oz. of gold $= 137\frac{1}{7}$.

10. Number of grains required = 39.6

$$\therefore \text{ cost} = \frac{39.6 \times \$17.50}{480}$$

$$= \$1.44375.$$

No. 7.

3. See H. Smith's Arithmetic, Art. 42.

5. 70000000000070000·0070000007

6. The word two should be three.

(1) Every number of four or more figures may be regarded as made up of a number of thousands plus the number of hundreds, tens, and units indicated by the figures in these places in the number. As 1000 is exactly divisible by 8 any number of thousands will be also divisible by 8, so if the number represented by the three right hand figures is exactly divisible by 8 the whole number must be divisible by 8.

(2)
$$3384 = 2^3 \times 3^2 \times 47,$$
$$8272 = 2^4 \times 11 \times 47,$$
$$7567 = 7 \times 23 \times 47 ;$$
$$\therefore \text{ H. C. F.} = 47 \text{ and}$$
$$\text{L. C. M.} = 2^4 \times 3^2 \times 7 \times 11 \times 23 \times 47$$
$$= 11582928.$$

(3)
$$8286604200 = 2^3 \times 3^2 \times 5^2 \times 7^2 \times 11 \times 13 \times 23 \times 29,$$
$$32340 = 2^2 \times 3 \times 5 \times 7^2 \times 11,$$
$$2522520 = 2^3 \times 3^2 \times 5 \times 7^2 \times 11 \times 13 ;$$

hence, as the other number must contain all the factors found

in the H. C. F. as well as those of the L. C. M. not found in the number given, the other number must be

$$2^2 \times 3 \times 5^2 \times 7^2 \times 11 \times 23 \times 29 = 107853900.$$

7. Elementary Arithmetic, Art. 141.

Since the denominator of a pure repetend of 3 figures is 999, the denominator must be 999 or a submultiple of this number, as 111 or 37.

8. (*a*) $\frac{1}{13} \times \frac{999999}{1000099} \times \frac{45}{1} = 3\,\frac{461238}{1000099}.$

(*b*) $\frac{213845}{285124} + \frac{2319}{2277} - \frac{1036832}{2150404} = 1\frac{33308121081563}{116341757170716}.$

9. (*a*)

$$1 = 1$$

$$\frac{1}{1 \cdot 2} = 0 \cdot 5$$

$$\frac{1}{1 \cdot 2 \cdot 3} = 0 \cdot 166666\dot{6}$$

$$\frac{1}{1 \cdot 2 \cdot 3 \cdots} = 1 \cdot 046666\dot{6}$$

$$\frac{1}{1 \cdot 2 \cdot 3 \cdots} = 0 \cdot 009333\dot{3}$$

$$\frac{1}{1 \cdot 2 \cdot 3 \cdots} = 0 \cdot 001555\dot{5}$$

$$\frac{1}{1 \cdot 2 \cdot 3 \cdots} = 0 \cdot 000222\dot{2}$$

$$\frac{1}{1 \cdot 2 \cdot 3 \cdots} = 0 \cdot 000027\dot{7}$$

$$\frac{1}{1 \cdot 2 \cdot 3 \cdots} = \frac{0 \cdot 00000308}{1 \cdot 724475}.$$

(*b*) \$21.75.

No. 8.

2. The simple rules deal with abstract numbers or with concrete numbers of one denomination. The compound rules deal with compounded numbers. (See Kirkland and Scott's Elem. Arith. Art. 100.)

3. $7689748 \times 999993 = 7689748 \times (1000000-7)$
$$= 7689748000000 - 53828236$$
$$= 7689694171764.$$

4. In reducing a vulgar fraction to a decimal the division is carried on by multiplying the numerator by 10, putting a figure in the quotient; multiplying the remainder by 10, &c. The factors of 10 being 2 and 5, it is evident that, when the denominator contains only 2's and 5's as factors, by multiplying by a sufficient number of 10's all the 2's and 5's in the denominator can be cancelled out so that only *one* will remain as the denominator, when the decimal must terminate. If, however, the denominator contains no 2's or 5's, there can be no cancelling and hence the decimal must begin to repeat at the decimal point; thus the decimal will be a pure repetend. If the denominator contains other factors than 2's or 5's, it is evident the decimal must begin to repeat after all the 2's or 5's have been cancelled; thus the decimal will be a mixed repetend.

5. (*a*) 7396.
   (*b*) $10{\cdot}0191$, $10{\cdot}0009$, $\cdot091091$, $\cdot00\dot{0}\dot{9}$, 1100.

6. 8 per cent.

7. (*b*) \$178.11.

8. $5\frac{1}{4}\%$(nearly.)

9. Amount of bond Jany. 1st 1882 $= 800 \times \$1.06 = \$848$

Sum paid for bond July 1st 1881 $= \$\dfrac{848}{1{\cdot}03\frac{1}{2}} = \$819\frac{67}{207}$

Proceeds from a note of \$1 for 93 days at $8\% = \$\frac{35756}{36500}$

$$\therefore \text{ face of the note} = \$(819\tfrac{67}{207} \times \tfrac{36500}{35756})$$
$$= \$836.37\ldots.$$

10. (*b*) 5 yards.

No. 9.

1. \$9840.

2.

$$
\begin{array}{r}
96879364852 \\
\underline{(144)(12)(1728)} \\
1162552378224 \\
13950628538688 \\
\underline{167407542464256} \\
1396242147001270425 6
\end{array}
$$

See Key to H. S. Arith., p. 1, for a similar example.

3. (*b*) \$14595.70.

5. A, B, and C do $\frac{23}{20}$ of the work.

It can readily be found that A does $\frac{10}{20}$, B $\frac{5}{20}$, and C $\frac{8}{20}$ of the work;

$$
\begin{aligned}
\therefore\ A \text{ gets } & \tfrac{10}{23} \text{ of } \$159.70 = \$69\tfrac{10}{23}; \\
B \quad \text{"} \ & \tfrac{5}{23} \text{ of } \$159.70 = \$34\tfrac{33}{46}; \\
C \quad \text{"} \ & \tfrac{8}{23} \text{ of } \$159.70 = \$55\tfrac{63}{115}.
\end{aligned}
$$

6. Selling price of a yard to gain 20% = \$6.84.

After the shrinkage each yard bought contained ($\frac{95}{100}$ of $\frac{195}{200}$ of $\frac{7}{4}$) sq. yd. = $\frac{5187}{3200}$ sq. yd.

Hence the selling price of 1 sq. yd. = $\frac{3200}{5187}$ of \$6.84.
= \$4.22 (nearly).

7. $\frac{25}{34}$ of a year.

8. Total income = ($\frac{30}{100}$ of $\frac{4}{90}+\frac{38}{100}$ of $\frac{9}{190}+\frac{32}{100}$ of $\frac{6}{105}$) of capital;

$\therefore$ ($\frac{30}{100}$ of $\frac{4}{90}+\frac{38}{100}$ of $\frac{9}{190}+\frac{32}{100}$ of $\frac{6}{105}$) of capital = \$1736$\frac{2}{3}$;
$\therefore$ Capital = \$35000.

9. 109.

10. 2 inches.

Algebra.--No. 1.

3. "When we multiply one integer a by another b, we may describe the operation thus: what we did with unity to obtain b we must now do with a to obtain b times a. To obtain b from unity *the unit* is repeated b times; therefore to obtain b times a the number a is repeated b times."—*Todhunter's Algebra, p. 62, or Lund's Companion to Wood's Algebra, p. 1.*

(1) $=\{(x^2+3)-2x\}\{(x^2+3)+2x\}=(x^2+3)^2-(2x)^2=$etc.

(2) $= x^{2m}-(2y^n+y^n)x^m+2y^{2n}=x^{2m}-3x^my^n+2y^{2n}$.

(3) $=\{x^{m^2}+(ax^m-b)\}\{x^{m^2}-(ax^m-b)\}=x^{2m^2}-(ax^m-b)^2=$ etc.

(4) $=(a^{\frac{1}{2}}+b^{\frac{3}{5}})(a^{\frac{1}{2}}+b^{\frac{3}{5}})(a^{\frac{1}{2}}-b^{\frac{3}{5}})=(a^{\frac{1}{2}}+b^{\frac{3}{5}})(a-b^{\frac{6}{5}})=$ etc.

4. (1) $=4x^4-64\div(x-2)=4(x^4-2^4)\div(x-2)=4(x^3.+x^22+x.2^2+2^3)$.

(2) Dividing by Horner's method we get, for quotient, $4.1x^2+2x-.6$.

(3) $(x^6+x^{-6}-2)\div(x^2+x^{-2}-2)$

$$=\left(\frac{x^3-x^{-3}}{x-x^{-1}}\right)^2=(x^2+1+x^{-2})^2.$$

5. See *Canada School Journal* for November, 1880, McLellan's Algebra, page 25, or Todhunter's Algebra, page 532.

6. Hamblin Smith's Algebra, page 70. $x+2$.

7. Hamblin Smith's Algebra, page 164. Arranging in order of indices we have $x^{6p}—4x^{4p}+4x^{2p}+6—12c^{-2p}+9x^{-6p}$. The square root of first term $=x^{3p}$, and since twice first into second must$=-4x^{4p}$; $\therefore$ second term must be $—2x^{p}$, and third term $=$square root of $9x^{-6p}$; hence square root$=x^{3p}—2x^{p}+3x^{-3p}$.

8. Hamblin Smith's Algebra, page 58.

(1) Equation $=x^{\frac{1}{2}}—1=4+\dfrac{x^{\frac{1}{2}}—1}{2}$; or $\dfrac{x^{\frac{1}{2}}—1}{2}=4$;

$\therefore \quad x=81.$

(2) The first equation may be written :—

$$c(ay+bx)=xy(x+y),$$

whence $c(ay+bx—xy)=xy(x+y—c)$;

dividing the second equation by this we have:

$$\frac{xy}{c}=\frac{abc}{xy}, \text{ or } xy=\pm c\sqrt{ab};$$

this value of xy being substituted in the given equations, we find $x=\sqrt{ac}$, and $y=\sqrt{bc}$

9. Let A's daily part$=\dfrac{1}{x}$, B's$=\dfrac{1}{y}$;

$$\therefore \quad \frac{1}{x}+\frac{1}{y}=\frac{1}{16};$$

and the remainder after 4 days$=\frac{3}{4}$;

$$\therefore \quad \frac{36}{y}=\frac{3}{4}, \text{ whence } y=48 \text{ days and } x=24 \text{ days.}$$

Algebra.—No. 2

1. See Hamblin Smith's Algebra, Arts. 266–272. See al o, Mathematical Notes in this month's EXAMINER.

2. See EXAMINER for June.

Let $(n-1)n(n+1)$ be the product of any three consecutive integers.

And $(m-1)m(m+1)$ be the product of any other three consecutive integers.

The difference of these products$=(n^2-1)n-(m^2-1)m.$

$$=n^3-m^3-(n-m),$$

which is evidently divisible by the difference of the middle integers.

3. $(x+a)(x+b)(x+c)(x+d)=x^4+(a+b+c+d)x^3$
$+(ab+ac+ad+bc+bd+cd)x^2+(abc+abd+acd+bcd)x+abcd.$
Coefficient of $x=2.6.10+2.6.14+2.10.14+6.10.14=1408.$

4. $(x+y+z)^3$

$$=x^3+y^3+z^3+3x^2(y+z)+3y^2(x+z)+3z^2(x+y)+6xyz\ ;$$

$$\therefore 1=1+3x^2(1-x)+3y^2(1-y)+3z^2(1-z)+6xyz$$

$$0=3(x^2+y^2+z^2)-3(x^3+y^3+z^3)+6xyz.)$$

$$=3-3+6xyz\ ;$$

$$\therefore\ xyz=0.$$

5. Dividend$=(a^2+b^2+c^2+d^2)x^2+(a^2+b^2+c^2+d^2)y^2$

$$=(a^2+b^2+c^2+d^2)(x^2+y^2).$$

Algebra.—*Continued.*

6. Hamblin Smith's Algebra, Art. 130.

$$(a+b)^3=a^3+b^3+3ab(a+b);$$

$$\therefore\ a^3+b^3+c^3-3abc=(a+b)^3+c^3-3ab(a+b)-3abc.$$

$$=(a+b)^3+c^3-3ab(a+b+c). \qquad (1)$$

$$a(a+2b)+b(b+2c)+c(c+2a)=a^2+b^2+c^2+2ab+2ac+2bc.$$

$$=(a+b+c)^2 \qquad (2)$$

The H. C. F. of (1) and (2) is evidently $a+b+c$.

7. Hamblin Smith's Algebra Arts. 145, 149, 158.

Adopting the meaning given in Art. 158, viz.: that $\frac{a}{b}$ represents the quotient resulting from the division of a by b, let $\frac{a}{b}=q$; then, since by the nature of division, a quotient, when there is no remainder, is such a quantity, that, if it be multiplied by the divisor, the product is the dividend. We have,

$$a=bq;$$

$$\text{and } ma=mbq;$$

$$\therefore\ \frac{ma}{mb}=q=\frac{a}{b}.$$

L. C. M. of denominators$=(a-b)(b-c)(c-a)$; adding, we have

$$\frac{(a+b)(a+c)(c-b)+(b+c)(b+a)(a-c)+(c+a)(c+b)(b-a)}{(a-b)(b-c)(c-a)}$$

In numerator let $a=b$, and it vanishes, therefore, $a-b$ is a factor; similarly $b-c$, and $c-a$ are factors; hence, numerator$=m(a-b)(b-c)(c-a)$. Letting $a=0$, $b=1$, $c=2$, we find $m=1$. The whole expression is therefore$=1$.

8. In the first case we divide by $a-a$, or 0, which does not necessarily give a correct result. It is frequently found, in mathematical reasoning, that the passage from x as a symbol of magnitude to x as a symbol not of magnitude,

but of the absence of all magnitude, is attended with consequences which require a special examination; it is not allowed to enter on any new ground without either establishing the accordance of the consequences of $x=0$ with those of $x=$some magnitude, or distinguishing and explaining the discrepancy, if any.

In the second case the two *zeros* are not necessarily equal; one may be infinitely less than the other.

9. An equation is true for certain values of the unknown quantity; an identity is true for *all* values of the unknown quantity.

Simplifying, x vanishes; hence, it must be true for *all* values of x, and is, therefore, an identity.

10. (1) Transpose, and we have

$$(a+x)^3+(b+x)^3+(c+x)^3-3(a+x)(b+x)(c+x)=0;$$

Comparing with the note to Ex. 6 we see that

$$(a+x+b+x+c+x) \text{ is a factor;}$$

$$\therefore\ 3x+a+b+c=0,$$

$$\text{or } x=-\frac{a+b+c}{3}.$$

(2) $$x=\frac{(b^2-c)^2bc+(c^2-a^2)ac+(a^2-b^2)ab}{(b-a)bc+(c-a)ac+(a-b)ab}=(a+b+c).$$

(3) Reduce each term to a mixed number, and 2 may be removed from each side, then each side is divisible by $a-b$, and the equation becomes

$$\frac{1}{x-a}-\frac{1}{x-b}=\frac{3}{x+a-2b}-\frac{3}{x+b-2a};$$

$$\text{whence } 9(x-a)(x-b)=(x+b-2a)(x+a-2b),$$

$$\text{and } 4(x-a)(x-b)=-(a-b)^2,$$

which gives two roots each equal to $\frac{1}{2}(a+b)$.

(4) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=$etc.; then $\frac{a}{b}=\frac{a+c+e+\&c.}{b+d+f+\&c.}$;
applying this principle, we have

$$\frac{x+y+z}{2(a+b+c)}=a+b+c\,;\ \therefore\ x+y+z=2(a+b+c)^2,$$

$$\text{and } x+y-z=(a+b+c)(b+c)\,;$$

from which z may easily be obtained, then the values of x and y may be written down by symmetry.

(5)
$$x^3+\frac{64}{x^3}=7x^{\frac{3}{2}}+\frac{56}{x^{\frac{3}{2}}}+2\,;$$

$$\therefore \left(x^{\frac{3}{2}}+\frac{8}{x^{\frac{3}{2}}}\right)^2-7\left(x^{\frac{3}{2}}+\frac{8}{x^{\frac{3}{2}}}\right)=18;$$

$$\therefore\ x^{\frac{3}{2}}+\frac{8}{x^{\frac{3}{2}}}=\frac{7\pm\sqrt{49+72}}{2}$$

$$=\frac{7\pm11}{2}=9 \text{ or } -2.$$

$$x^{\frac{3}{2}}+\frac{8}{x^{\frac{3}{2}}}=9;$$

$$\therefore\ x^3-9x^{\frac{3}{2}}=-8\,;$$

$$x^{\frac{3}{2}}=\frac{9\pm\sqrt{81-32}}{2}$$

$$=\frac{9\pm7}{2}=8 \text{ or } 1:$$

$$\therefore\ x^{\frac{1}{2}}=2 \text{ or } 1$$

$$x=4 \text{ or } 1.$$

The remaining roots may be found by equating $x^{\frac{3}{2}}+\frac{8}{x^{\frac{3}{2}}}$ to -2

No. 3.

1. $a^m - a^n = a^n\left(a^{m-n} - 1\right)$. Now if $m-n$ is even, $a^{m-n} - 1$, is divisible by $a+1$. The other cases may be proved in a similar manner.

2. Take for example the expression x^3+px^2+qx+r, and let

$$\frac{x^3+px^2+qx+r}{x+1} = Q + \frac{R}{x+1}$$

where R does not contain x, and will, therefore, remain the same, whatever value may be given to x. Let $x=-1$; then $-1+p-q+r=R$. Now if the expression is divisible by $x+1$, $R=0$; therefore $p+r=q+1$; that is, *the expression x^3+px^2+qx+r, is divisible by $x+1$, if the sum of the alternate coefficients of the dividend are equal and of opposite signs.*

In a similar manner it may be shown that the expression is divisible by $x-1$, when the sum of the coefficients of the dividend are equal to 0.

These principles are sometimes useful in finding by inspection one of the roots of an equation.

In the example given in question 2, the sums of the alternate terms are 2 and -2 respectively; it is, therefore, divisible by $x+1$. Also the sum of all the coefficients $=0$, it is, therefore, divisible by $x-1$.

3. If $x-y$ is the *H. C. D.* of two algebraical expressions, so also is $y-x$, and each may be made numerically greater than the other by assigning certain values to x and y.

4. By means of the formula $x^3-y^3=(x-y)(x^2+xy+y^2)$, the expression reduces to

$$\frac{9(b-c)\ (a^2+b^2+c^2-ab-bc-ca)}{(b+c-2a)\ (a^2+b^2+c^3-ab-bc-ca)}$$

5. The square roots are $\frac{a}{3}-\frac{x}{7}-\frac{y}{4}$, $\frac{y}{4}+\frac{x}{7}-\frac{a}{3}$.

The results may be expressed by $\pm\left\{\frac{a}{3}-\frac{x}{7}-\frac{y}{4}\right\}$

The student will notice that the roots may easily be obtained by inspection. The square root of the first term is $\frac{a}{3}$, and $-\frac{2ax}{21}\div\frac{2a}{3}=-\frac{x}{7}$ The square root of $\frac{y^2}{16}$ is the last term, and $\frac{2a}{3}\times\frac{y}{4}$ must equal $-\frac{ay}{6}$; $\therefore$ last term must be *minus.*

6. Let a and $a+1$, be any two consecutive numbers:

then, $\quad (a+\overline{a+1})^2=(2a+1)^2=4a^2+4a+1$, which $=4a\,(a+1)+1$, expresses the theorem.

$$19^2=(9+10)^2=4\times9\times10+1=360+1=361.$$
$$23^2=(11+12)^2=4\times11\times12+1=528+1=529$$

7. These equations present no difficulties.

8.
$$\frac{x^2+(a+b)\,x+ab}{x+a+b}=\frac{x^2+(c+d)\,x+cd}{x+c+d},$$
$$x+\frac{ab}{x+a+b}=x+\frac{cd}{x+c+d};$$
$$\therefore \quad x=\frac{(a+b)\,cd-(c+d)\,ab}{ab-cd}$$

Here $\quad ax+cy+bz=cx+by+az;$

$\therefore \quad (a-c)\,x+(c-b)\,y+(b-a)\,z=0.$

By symmetry $\quad (c-b)\,x+(b-a)\,y+(a-c)\,z=0.$

By elimination, we have—

$$y=\frac{(a-c)^2-(c-b)\,(b-a)}{(b-a)^2-(c-b)\,(a-c)}\,x=\frac{a^2+b^2+c^2-ab-ac-bc}{a^2+b^2+c^2-ab-ac-bc}\,x.$$
$$=x=z, \text{ by symmetry.}$$

But $\quad ax+cy+bz=a^3+b^3+c^3-3abc.$

$bx+ay+cz=$ " "

$cx+by+az=$ " "

Adding these equations, we have—

$(a+b+c)\,(x+y+z)=3(a^3+b^3+c^3-3abc)$;

$\therefore \qquad x+y+z=3(a^2+b^2+c^2-ab-ac-bc),$

Since $\qquad x=y=z,\ 3x=3(a^2-b^2+c^2-ab-ac-bc);$

$\therefore \qquad x=a^2+b^2+c^2-ab-ac-bc=y=z.$

The interest of \$1 in 4 per cents. is \$ $\frac{4}{96}=\frac{1}{24}$,

" \$1 " 5 " \$$\frac{5}{100}=\frac{1}{21}$.

Let \$$x$ be invested in the 4 per cents, and

\$$y$ " " 5 "

Then $\qquad \frac{x}{24}+\frac{y}{21}=\frac{4\frac{1}{2}}{100}(x+y);$

$\therefore \qquad 14x=11y,$

or $\qquad \frac{x}{y}=\frac{11}{14};$

$\therefore$ For every \$11 he invests in the four per cents., he must invest \$14 in the five per cents.

10 The equation reduced becomes

$$(a+b+c)x^3+(a^2+b^2+c^2)x^2-3abcx-abc(a+b+c)=0.$$

But since $a+b+c=0$ is a condition, we have

$(a^2+b^2+c^2)x^2-3abcx=0$, which gives

$x=0$, and $x=\frac{3abc}{a^2+b^2+c^2}$

No. 4.

1. It is obvious that $bx+cy$ must be one of the factors, and that $x^2+axy+y^2$ must be divisible by $bx+cy$. After performing the division the remainder will be $\left\{1-\frac{abc-c^2}{b^2}\right\}y^2$, which must be equal to zero, in order that the division may leave no remainder.

2. If any function of x, say, x^3+px^2+qx+r be divided by $x-a$.

The coefficients of the dividend are, 1, p, q, r,

" quotient are, 1, $a+p$, a^2+ap+q,

and the remainder is $\qquad a^3+pa^2+qa+r$.

Divide $\qquad x^3-6x^2+11x-6$ by $x-3$.

Coeffs. of dividend $1, -6, +11, -6$;
now, multiplying each of these by 3, and adding the succeeding one, we have,

$$1,\ 3-6,\ -9+11,\ 0,$$

or $1, -3, 2$, the coeffs. of the quotient.

The remainder is $27-54+33-6=0$.

In the question, the coeffs. of dividend are, $1, -p, q, -r, s, -t$,
" " quotient are, $1, a-p, a^2-ap+q$, $a^3-a^2p+aq-r$, $a^4-a^3p+a^2q-ar+s$,
and the remainder is $a^5-pa^4+qa^3-ra^2+sa-t$.

In the sec'd case, the coeffs. of the dividend are,
$1, -8, 12, -18, 20, -30$;
Therefore, the coeffs. of the quotient are,
$1, -4, -4, -34, -116$.
The remainder is obtained by writing 4, instead of x.

$$4^5-8.4^4+12.4^3-18.4^2+20.4-30=-494.$$

3. As the common divisor is a quadratic factor, the remainder will be of the form $px+q$, and must be zero, and as x is not zero, we must have $p=0$, and $q=0$.

Now divide cx^5+bx^4+a by ax^5+bx+c in the usual way; the second remainder is

$$cx^2+\frac{c^2+b^2-a^2}{b}x+c\ ;$$

let $\frac{c^2+b^2-a^2}{b}=m$, and continue the process, as in the method of finding the H. C. F. and we get for remainder,

$$\left\{\frac{bm}{c}-\frac{bc}{a}-\frac{m(m^2-bc^2)}{c^3}\right\}x+\frac{a^2-c^2}{a}-\frac{m^2-bc^2}{c^2} \text{ which}$$

must $=0$;

$$\therefore\quad \frac{a^2-c^2}{a}=\frac{m^2-bc^2}{c^2}$$

$$\frac{mab-bc^2}{ac}=\frac{m(m^2-bc^2)}{c^3};$$

$$\therefore\quad \frac{a^2-c^2}{mab-bc^2}=\frac{1}{m}, \text{ or } (a^2-c^2-ab)m=bc^2\ ;$$

Substituting the value of m we have $b^2c^2=(c^2-a^2+b^2)$ (c^2-a^2+ab).

4. In Arithmetic a fraction is any part or parts of a unit or whole ; in Algebra, any quantity $\frac{a}{b}$ is called a fraction, although a and b are not necessarily representatives of whole numbers, as thoy would require to be if the fraction is an arithmetical fraction. The Algebraical fraction $\frac{a}{b}$ just means that any quantity affected by it is to be multiplied by a and divided by b. This definition, however, through the greater generality of Algebra, includes that of an Arithmetieal fraction.

5. In the second term of the denominator, $-3bc$ should be $+3bc$.

$$\text{Fraction}=\frac{(b+c)\{a^2(b-c)-ab(2b-c)+b^3\}}{a(b+c)\{a^2(b+c)-ab(2b+c)+b^3\}}$$
$$=\frac{(b+c)(a-b)(ab-ac-b^2)}{a(b+c)(a-b\ (ab-ae-b^2)}$$

6. If $a>b$, then $a-b>0$, and $a^2-2ab+b^2>0$,

$\therefore\quad a^2+b^2>2ab$;

$\therefore\quad \frac{a}{b}+\frac{b}{a}>2.$

7. In the complete square $a^2+2ab+b^2$, we see that 4 times the product of the extremes is equal to the square of the mean ; hence $4acx^2=b^2x^2$, or $b^2=4ac$.

We may obtain the same result by extracting the square root, and equating the remainder to zero ; or we may proceed thus :—If it be a complete square it is

$$=\left(a^{\frac{1}{2}}x+c^{\frac{1}{2}}\right)^2=ax^2+2a^{\frac{1}{2}}c^{\frac{1}{2}}x+c;$$

$$\therefore\qquad 2a^{\frac{1}{2}}c^{\frac{1}{2}}=b,\text{ or } b^2=4ac,\text{ as before.}$$

The difference is $x^6+6x^5+(12-a)x^4+(56-b)x^3+(9b-c)x^2+96x+64$.

It is obvious that x^6, $6x^5$, and 64, are the first, second, and fourth terms of the square.

Suppose m to be the coeff. of the third term, then,

$$x^6+6x^5+(12-a)x^4+(56-b)x^3+(96-c)x^2+96x+64$$
$$=(x^3+3x^2+mx+8)^2.$$

Expanding and equating coeffts, we get $m=6$, $a=-9$, $b=-4$, and $c=-12$.

8. (1.) $a+b+c=0,$

(2.) $-ab-ac+bc=0,$

(3.) $ab+ac+bc=b+c-a$;

$$2bc=b+c-a,$$
$$0=a+b+c$$
$$\therefore \quad -bc=a.$$

10. (2.) $7-4\sqrt{3}=(2-\sqrt{3})^2.$

Hence equation becomes $\{(2-\sqrt{3})x\}^2+(2-\sqrt{3})x=2.$

(3.) Multiply the second equation by 3 and add to the first equation.

$$(x+y)^3-9(x-y)=0\ ;$$
$$(x+y)^2=9 \text{ or } x+y=+3\ ;$$
$$\text{Also} \quad xy(x+y)=2, \text{ etc.}$$

No. 5.

1. $\{\frac{1}{2}(a+b)\}^2-\{\frac{1}{2}(a-b)\}^2=ab.$ The product of any two numbers is equal to the square of half their difference subtracted from the square of half their sum.

2. Since $a^2-b^2=(a+b)(a-b),$

$$(3x^2-4x+2)^2-(2x^2+9x+3)^2=(5x^2+5x+5)(x^2-13x-1)$$

which is evidently divisible by x^2+x+1.

3. $x^n-na^{n-1}x+(n-1)a^n=(x^n-a^n)-na^{n-1}x(x-a)$

$$=(x-a)\{x^{n-1}+x^{n-2}a+x^{n-3}a^2+\ldots\ldots+a^{n-1}-na^{n-1}\}$$
$$=(x-a)\{(x^{n-1}-a^{n-1})+a(x^{n-2}-a^{n-2})+\&c. \text{ to } n \text{ terms}\}$$

which is evidently divisible by $(x-a)(x-a)$ or $(x-a)^2$.

$$x^5-5a^4x+4a^5=x^5-5a^4x+5a^5-a^5$$
$$=x^5-a^5-5a^4(x-a)$$
$$=(x-a)(x^4+ax^3+a^2x^2+a^3x+a^4-5a^4)$$
$$=(x-a)\{x^4-a^4+a(x^3-a^3)+a^2(x^2-a^2)+a^3(x-a)\}$$
$$=(x-a)^2\{x^3+ax^2+a^2x+a^3+a(x^2+ax+a^2)+a^2(x+a)+a^3\}$$
$$=(x-a)^2\{x^3+2ax^2+3a^2x+3a^3\}.$$

5. If x^2+ax+b be divided by $x+c$, the quotient is $x+a-c$, and the remainder $b-(a-c)c=0$, and therefore $b=c(a-c)$.

If x^2+mx+a be also divided by $x+c$, the quotient is $x+m-c$ and the remainder $a-c(m-c)=0$

The product of x^2+ax+b and $x+m-c$ will be least common multiple. If $c(a-c)$ be substituted for b, the required form will be obtained.

6. $$\frac{(ac+bd)^2}{(a^2+b^2)(c^2+d^2)}=\frac{(ac+bd)^2}{(ac+bd)^2+(ad-bc)^2},$$ and since $(ad-bc)^2$ is always positive and therefore >0; the den'r is always $>$ num'r.

7. The L.C.M. is $(a-b)(b-c)(c-a)$; adding in the usual manner, we have

$$\frac{(x-b)(x-c)(c-b)+(x-c)(x-a)(a-c)+(x-a)(x-b)(b-a).}{(a-b)(b-c)(c-a)}$$

The num'r does not vanish for $x=0$; $\therefore$ x is not a factor.

" " " $a=0$; $\therefore$ a is not a factor, and by symmetry, neither b nor c is a factor. Let $a=b$, and the num'r vanishes; $\therefore$ $a-b$ is a factor; similiarly $b-c$ and $c-a$ are factors, and since the expression is of three dimensions the only factors that we can have are $(a-b)$ $(b-c)$, and $(c-a)$; the numerator must therefore $=m(a-b)(b-c)(c-a)$. To find m let $x=0$, $a=1$, and $b=2$, and $c=3$ and m is found to be $=1$. Therefore the expression in its simplest form $=1$.

8. (1) Equation $=-4+\dfrac{18}{3x+1}+2-\dfrac{12}{2x+7}$

$$=-9+\frac{19}{x+1}+7-\frac{19}{x+2};$$

whence $$\frac{114}{(3x+1)(2x+7)}=\frac{19}{(x+1)(x+2)},\ \text{\&c.}$$

(2) Multiply each side of the equation by $4(x+2)^{\frac{3}{2}}$, and we have $8+2(x+2)^2=17(x+2)$.

9. Hamblin Smith's Algebra, Art. 326, where it is shown that a quadratic equation cannot have more than two roots.

If the equation is satisfied by more than two values of x, it is no longer an equation but an identity, and, therefore, true for *all* values of x.

By solving the equations it will readily be seen that their roots are equal in magnitude and opposite in sign.

In the equation $ax^2+bx+c=0$; change x into $\frac{1}{x}$ and we have $cx^2+bx+a=0$, whose roots are the reciprocals of the roots of the equation $ax^2+bx+c=0$.

10. Let x=width of path, then $a-2x$, $b-2x$ are the sides of the inner rectangle,

$$ab=\text{area of path and inner rectangle,}$$
$$(a-2x)(b-2x)=\text{area of inner rectangle ;}$$
$$\therefore \quad ab-(a-2x)(b-2x)=\text{area of path}=\frac{(a-2x)(b-2x)}{m}$$

No. 6.

1. See note at end of EXAMINER for June.

When $ax^2+bxy+cy^2$ is divided by $x+m$, the remainder $=a(-m)^2-bmy+cy^2$; if the quantity is divisible this remainder must$=0$; equate it to *zero* and find y.

2. See Hamblin Smith's Algebra, Art. 58; Todhunter's Larger Algebra, Chap. V.

The subject will be discussed in a future number of the EXAMINER.

3. (1) $$x^2+a^2=\frac{x^4-a^4}{x^2-a^2}=\frac{x^{2^2}-a^{2^2}}{x^2-a^2}$$

$$x^4+a^4=\frac{x^8-a^8}{x^4-a^4}=\frac{x^{2^3}-a^{2^3}}{x^{2^2}-a^{2^2}}$$

$$\&c., \quad = \quad \&c.$$

$$x^{2^n}+a^{2^n}=\ldots\ldots=\frac{x^{2^{n+1}}-a^{2^{n+1}}}{x^{2^n}-a^{2^n}}$$

Multiplying, we have—

$$(x^2+a^2)(x^4+a^4)\ldots\ldots(x^{2^n}+a^{2^n}=\frac{x^{2^{n+1}}-a^{2^{n+1}}}{x^2-a^2}$$

(2) Product of 2 factors $=a^2-a^2$,

$$\begin{array}{llll} \text{"} & \text{"}\ 3 & \text{"} & =a^4-b^4=a^{2^2}-b^{2^2} \\ \text{"} & \text{"}\ 4 & \text{"} & \ \ a^8-b^8=a^{2^3}-b^{2^3} \\ \ldots\ldots\ldots & & & =\ldots\ldots\ldots \\ & \text{"}n+1 & \text{"} & =a^{2^n}-b^{2^n} \end{array}$$

4. $2x+5y+3z=(2x+3y)+(2y+3z)$;

hence, $\dfrac{(2x+3y)^3+(2y+3z)^3}{(2x+3y)+(2y+3z)}=(2x+3y)^2-(2x+3y)(2y+3z)+(2y+3z)^2$

5. Hamblin Smith's Algebra, Arts. 127, 168.

The following method of finding the H. C. F. is often easier than the ordinary method :

Write the coeffs., inserting 0's where powers of x are wanting.

2	0	0	−11	0	−9	(1)
4	11	0	0	0	81	(2)
18	0	0	−99	0	−81	..(1)×9..(3)
22	11	0	−99			....(2)+(3)
2	1	0	−9			................(4)
4	11	0	0	0	+81	..........(2)
4	0	0	−22	0	−18	..(1)×2..(5)
	11	0	22	0	99	..(2)−(5). (6)
	1	0	2	0	9	..(6)÷11..(7)
		2	1	0	−9	..(4)
	1	2	3			......(4)+(7)
	x^2+2x+3,					

and since this expression cannot be resolved into simpler factors it must be the Highest Common Factor. *See McLellan's Algebra.*

6. x^4-1 is the H.C.F. of the denominators ;

hence, $\dfrac{x}{x^2-1}+\dfrac{x^2+x-1}{x^3-x^2+x-1}+\dfrac{x^2-x-1}{x^3+x^2+x+1}-\dfrac{x^3}{x^4+1}$

$$=\frac{x(2x^2+1)}{x^4-1}.$$

Change x into $\dfrac{a}{b}$, and simplify ; we have—

$$\frac{ab}{a^2-b^2}+\frac{a^2b+ab^2-b^3}{a^3-a^2b+ab^2-b^3}+\frac{a^2b-ab^2-b^3}{a^3+a^2b+ab^2+b^3}-\frac{a^3b}{a^4-b^4}$$

$$=\frac{ab(2a^2+b^2)}{a^4-b^4};$$

$\therefore$ dividing by b, value of expresssion required $=$ $\dfrac{a(2a^2+b^2)}{a^4-b^4}$.

7. Hamblin Smith's Algebra, Art. 223.

Expression $=(a^2+b^2+c^2+d^2)^2$; $\therefore$ sq.rt. $=a^2+b^2+c^2+d^2$,

8. $m=4$, will be an identity, since in that case, the x's vanish.

No value of m can make the expression an equation.

9. (1) $\dfrac{x+7-5}{x+7}-\dfrac{x+5-7}{x+5}=\dfrac{3x+17}{x^2+12x+35}$

$$\frac{-5}{x+7}+\frac{7}{x+5}=\ldots\ldots\ldots$$

$$\frac{2x+24}{x^2+12x+35}=\frac{3x+17}{x^2+12x+35};$$

$$\therefore x=7.$$

(2)

$$\left.\begin{aligned} 3xy&=x+y,\\ xz&=x+z,\\ yz&=2y+z;\end{aligned}\right\}\begin{aligned}&\ldots\ldots(1)\\&\ldots\ldots(2)\\&\ldots\ldots(3)\end{aligned}$$

Dividing (1) by xy, (2) by xz, and (3) by yz, we have—

$$3=\frac{1}{y}+\frac{1}{x},\quad\ldots\ldots\ldots(4)$$

$$1=\frac{1}{z}+\frac{1}{x},\quad\ldots\ldots\ldots(5)$$

$$1=\frac{2}{z}+\frac{1}{y}\ ; \ldots\ldots\ldots(6)$$

$$\therefore 2=\frac{1}{y}-\frac{1}{z}\ldots(4)—(5)\ldots(7)$$

$$-\ 1=\frac{3}{z}\ldots\ (6)—(7)$$

$$\therefore z=-3.\ \&c.,$$

(3) When reduced the equation becomes—

$$-(a+x)(a-c+x)=(b-c+x)(b+x).$$

(4) $x=\dfrac{c_1b_2-b_1c_2}{a_1b_2-b_1a_2}$; $y=\dfrac{c_1a_2-a_1c_2}{b_1a_2-a_1b_2}$

If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, then $a_1b_2=a_2b_1$, $a_1c_2=a_2c_1$, $b_1c_2=o_2c_1$; in this case, x and y assume the form $\frac{0}{0}$, and their values are *indeterminate.* It will be found, in fact, that, in this case, there are not two independent equations, one of them being only a multiple of the other ; for let $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=m$; then $a_1=ma_2$, $b_1=mb_2$, $c_1=mc_2$; and, therefore, the first equation becomes $m(a_2x+b_1x)=mc_2$, which is identical with the second equation. There being then only one equation between x and y, if we give *any* value to x or y, there will be a corresponding value for y or x.

10. $\sqrt{x^2+7x+6}=\sqrt{x^2+4x-5}-2$;

Squaring and simplifying, we have—

$$7x^2+22x=129\ ;$$

$$\therefore x=\ 3 \text{ or} -6\tfrac{1}{7}.$$

If 3 be substituted for x, it does not satisfy the conditions of the equation. The reason is that, x^2+7x+6, and x^2+4x-5, are the squares of $-\sqrt{x^2+7x+6}$, and $-\sqrt{x^2+4x-5}$, as well as of the same quantities with + signs ; thus squaring both sides of the equations introduces a new condition, and a

new value of the unknown quantity corresponding to it, which had no place before. Here 3 is the value which corresponds to the supposition that

$$-\sqrt{x^2+7x+6}=-\sqrt{x^2+4x-5}-2.$$

It should be particularly remembered, that since $+a\times+b$ is equal to $-a\times-b$, in the multiplication and evolution of quantities new values are always introduced, which, if not again excluded by the nature of the question, will appear in the final equation.

No. 7.

1. $+(a-3b+3c)$, and $-(3b-a--3c)$.

2. $(a_1+a_2+a_3-a_4)x+(a_1+a_2+a_4-a_3)y+(a_1+a_3+a_4-a_2)z$.

3. Answered in a preceding number.

4. Multiply the quantities together and in the product make such substitution as will make each of the three factors identical to $x+y$.

5. $x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5=\dfrac{x^6-y^6}{x-y}$;

divide this expression by x^3+y^3, we obtain the required ex pression:

$$\frac{x^6-y^6}{(x-y)(x^3+y^3)}=\frac{x^3-y^3}{x-y}=x^2+xy+y^2.$$

6. $\dfrac{a^{mn}-b^{mn}}{a^m-b^m}=a^n+a^{n-m}b^m+a^{n-2m}b^{2m}+\ldots\ldots+a^{n-\overline{p-1}m}b^{\overline{p-1}m}$,

7. $$\frac{x^3+px^2+px+1}{x+1}=\frac{(x^3+1)+px(x+1)}{x+1}$$
$$=x^2-x+1+px=x^2+(p-1)x+1.$$

$x^5+px^4+qx^3+qx^2+px+1=(x^5+1)+px(x^3+1)+qx^2(x+1)$, which is divisible by $x+1$.

8. Put $a=-b$ and the expression becomes *zero*. Therefore, $a+b$ is a factor, and from symmetry, $b+c$, and $c+a$ must also be factors; and since the expression is of three dimensions, the factors $=m(a+b)(b+c)(c+a)$ where m is a numerical quantity.

To find m; in the identity—
$(a+b+c)^3-(a^3+b^3+c^3)=m(a+b)(b+c)(c+a)$ put $a=b=1$, and $c=0$, and m is found $=3$.

9. $(ay+bx)^2=a^2$,
$(by-ax)^2=b^2$;
$\therefore\ (a^2+b^2)y^2+(a^2+b^2)x^2=a^2+b^2$;
$\therefore\qquad x^2+y^2=1.$

10. (1) $+abe$, should be $-abc$.

Expression $=b(x^2+\overline{c-a}x-ac)+x(x^2+\overline{c-a}x-ac)$
$=(x+b)(x+c)(x-a)$.

(2) The nearest complete square contained in the expression is x^2+3x-2.

$(x^2+3x-2)^2=x^4+6x^3+5x-12x+4$
$=x^4+6x^3-11x-12x+4+16x^2$;

$\therefore\ (x^2+3x-2)-(4x)^2=x^4+6x^3-11x-12x+4$;
and $(x^2+3x-2)^2-(4x)^2=(x^2+7x-2)(x^2-x-2)$.

(3) $-b(y^2-2ay+a^2)+y(y^2-2ay+a^2)$
$=(y-a)^2(y-b)$.

No. 8.

2. Hamblin Smith's Algebra, Articles 267, 271, 272.

3. The quantities must be arranged with indices, either ascending or descending.

4. $3x^3-2x^2-5x-3$. See McLellan's Algebra, page 26, Ex. 5.

$$x^2+x^{-2}+1=x^2+2+x^{-1}-1=(x+x^{-1})^2-1$$
$$=(x+x^{-1}+1)(x-1+x^{-1});$$

therefore $x^2+x^{-2}+1$ divided by $x-x^{-1}+1=x-1+x^{-1}$.

5. The quotient is $ax+b$, the remainder $(c-ab^2)a+(d-b^3)$, of which each partmust be equal to zero,

$$\text{or} \quad c-ab^2=0,$$
$$\text{and} \quad d-b^3=0,$$
$$\text{whence } ad=bc.$$

6. Put $a=0$, and the expression becomes zero, therefore a is a factor, and from symmetry b and c are factors. Now the product abc is only of *three* dimensions and the expression itself is of *four* dimensions, there must, therefore, be another factor of *one* dimension, and this can only be $a+b+c$. The 12 is found in the usual manner.

7. Hamblin Smith's Algebra, Article 168.

$$2x^{\frac{3}{2}}(x^{\frac{1}{2}}+1)-(x-1), \quad x^{\frac{3}{2}}(x^{\frac{1}{2}}+1)-2(x-1).$$

The H. C. F. is evidently $x^{\frac{1}{2}}+1$; hence, the L.C.M. is

$$\{2x^{\frac{3}{2}}-(x^{\frac{1}{2}}-1)\}\{x^{\frac{3}{2}}(x^{\frac{1}{2}}+1)-2(x-1)\}.$$

8. $$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots=x.$$

$$\therefore \; a=bx, \qquad \text{Again, } a^n=b^nx^n,$$
$$c=dx, \qquad c^n=d^nx^n,$$
$$e=fx, \qquad e^n=f^nx^n,$$
$$\text{etc.}=\text{etc.}; \qquad \text{etc.}=\text{etc.};$$
$$\frac{ace\ldots.}{bdf\ldots.}=x^n. \qquad \therefore \frac{a^n+c^n+e^n+\ldots.}{b^n+d^n+e^n+\ldots.}=x^n;$$

$$\therefore \frac{ace\ldots.}{bdf\ldots.}=\frac{a^n+c^n+e^n+\ldots.}{b^n+d^n+e^n+\ldots.}$$

9. Since a, b, c are sides of a triangle,

$$\begin{aligned} a+b&>c, & \therefore\ ac+bc&>c^2 \\ b+c&>a, & ab+ac&>a^2 \\ c+a&>b, & bc+ab&>b^2; \\ & & \therefore\ a^2+b^2+c^2&<2(ab+bc+ca). \end{aligned}$$

Again since any positive quantity is greater than 0, we have

$$\begin{aligned} (a-b)^2>0, \text{ or } a^2+b^2&>2ab \\ b^2+c^2&>2bc \\ c^2+a^2&>2ca; \\ \therefore\ a^2+b^2+c^2&>ab+bc+ac. \end{aligned}$$

10. (1) $\dfrac{x-1+2}{x-1}+\dfrac{x-2+4}{x-2}=2\ \dfrac{11x-18+36}{11x-18}$;

$$1+\frac{2}{x-1}+1+\frac{4}{x-2}=2\left(1+\frac{36}{11x-18}\right)$$

$\dfrac{1}{x-1}+\dfrac{2}{x-2}=\dfrac{36}{11x-18}$, which can be solved by the ordinary methods.

(2) $\sqrt{x}=\sqrt{x+1}-\sqrt{x-1}$; square and simplify.

(3) $\dfrac{\sqrt{x^4+1}}{x^2-1}=\dfrac{\sqrt{2a^2+2b^2}}{2b}$,

$$\frac{x^4+1}{x^4-2x^2+1}=\frac{2a^2+2b^2}{4b^2},$$

$$\frac{x^4+1}{2x^2}=\frac{2a^2+2b^2}{2a^2-2b^2}=\frac{a^2+b^2}{a^2-b^2},$$

$$\frac{x^4+2x^2+1}{x^4-2x^2+1}=\frac{a^2}{b^2},$$

$$\frac{x^2+1}{x^2-1}=\frac{a}{b},$$

$$x=\sqrt{\frac{a+b}{a-b}}.$$

No. 9.

1. A negative quantity implies that it is of a contrary nature in some way to what it would have been if positive. We cannot, therefore, interpret its meaning independently of a knowledge of the corresponding positive quantity.

2. $-x^3+8x^2y+7xy^2+7y^3$.

3. (*a*) $-21b$; (*b*) $2a^2-3ab+4b^2$.

4. (1) Write $-2a$ for $b+c$, or $-(2a+c)$ for b, and the expression becomes zero; $2a+b+c$ is therefore a factor, and by symmetry $2b+c+a$, and $2c+a+b$, are also factors, and since the expression is only of three dimensions, these are all the factors. The expression is, therefore, equal to

$$m(2a+b+c)(2b+c+a)(2c+a+b);$$

put $a=b=0$, and $c=1$, and m will be found $=3$.

(2) Similar to the above.

5. Hamblin Smith's Algebra, Art. 128.

(1) $x^4+p^2x^2+p^4=x^4+2p^2x^2+p^4-p^2x^2=(x^2+p^2)^2-(px)^2$
$$=(x^2+px+p^2)(x^2-px+p^2),$$
and $x^4+2px^3+p^2x^2-p^4=(x^2+px)^2-(p^2)^2$
$$=(x^2+px+p^2)(x^2+px-p^2);$$

the H. C. F. is, therefore, x^2+px+p^2.

(2) For the method of finding the L. C. M. of fractions see Hamblin Smith's Arithmetic, Art. 81. In this case find the L C. M. of mixed numbers and the algebraical quantities separately.

6. Hamblin Smith's Algebra, Art. 148.

$$(1)\ 1-\frac{a^2+b^2-c^2}{2ab}=\frac{2ab-a^2-b^2+c^2}{2ab}=\frac{c^2-(a-b)^2}{2ab}$$
$$=\frac{(c+a-b)(c-a+b)}{2ab},$$

which being = last term, expression $=0$.

(2) $$\frac{(x^n)^3-1}{x^n-1}=(x^n)^2+x^n+1,$$

and $$\frac{(x^n)^2-1}{x^n+1}=x^n-1;$$

$$\therefore\ \frac{x^{3n}-1}{x^n-1}-\frac{x^{2n}-1}{x^n+1}=x^{2n}+2.$$

(3) Resolve into factors and simplify. Result$=1$.

7. (1) Expression$=x^2-2+\frac{1}{x^2}+2\left(x-\frac{1}{x}\right)-1+2.$

$$=\left(x-\frac{1}{x}\right)^2+2\left(x-\frac{1}{x}\right)+1,$$

the square root of which is evidently$=x-\frac{1}{x}+1$.

(2) Expression$=\frac{x^2}{y^2}+2+\frac{y^2}{x^2}-\frac{2}{\sqrt{2}}\left(\frac{x}{y}+\frac{y}{x}\right)+\left(\frac{1}{\sqrt{2}}\right)^2,$

the square root of which is$=\frac{x}{y}+\frac{y}{x}-\frac{1}{\sqrt{2}}\cdot$

8. Dividing the numerators by the corresponding denominators, we see that $x+y+z$ is a factor of each term, and, therefore, when $x+y+z=0$, the whole$=0$.

9. (1) Dividing numerators by denominators, and then dividing by 2, we have—

$$\frac{1}{x+1}-\frac{1}{x+2}+\frac{1}{x-4}=-\frac{x+3}{x^2-9},$$
$$=-\frac{1}{x-3};$$

from which we find $x=1$.

(2) Taking the reciprocals of each equation and dividing, we have—

$$\frac{1}{y}+\frac{2}{x}=\frac{5}{2},\quad \frac{2}{z}+\frac{3}{y}=\frac{13}{6},\quad \frac{3}{z}+\frac{4}{x}=\frac{13}{3},$$

from which x, y, z can be found by the ordinary method of elimination.

10. Let x=the distance, $\frac{x-\frac{1}{2}}{3\frac{1}{2}}$=hours of A, $\frac{x+\frac{1}{2}}{4\frac{1}{2}}$ =hours of B, but $\frac{2x-1}{7}=\frac{2x+1}{9}+\frac{2}{3}$, whence $2x=29$ miles.

Statics and Hydrostatics.--No. 1.

NOTE.—The references are to Kirkland's Elementary Statics.

1. Statics, pages 4 and 7.

2. Statics, page 168.

Let a be one force, and $2a$ the other, then as in ex. 2, page 16, of Statics, we have,

$$R^2=\left(2a-\frac{a}{2}\right)^2+\left(\frac{a}{2}\sqrt{3}\right)^2=3a^2; \therefore R=a\sqrt{3}.$$

3. The converse of the Polygon of forces. Form a quadrilateral, of which the given pressure is one side. Any three lines equal in magnitude and parallel in direction to the other sides, will be the components.

4. If the resultant be reversed the three forces are in equilibrium. Resolve all the forces *vertically* and *horizontally* and we have

$$\tfrac{1}{2}P\sqrt{2}-\tfrac{1}{2}Q=15$$
$$\tfrac{1}{2}P\sqrt{2}+\tfrac{1}{2}Q\sqrt{3}=15\sqrt{3}.$$

from which P and Q can easily be determined.

5. Resolve the weight of beam AB into 50 lbs. acting at A, and 50 lbs. acting at B, and the weight of the beam B into 50 lbs. acting at C, and 50 lbs. acting at B.

Let P be the force acting along the beam AB, and Q the force acting along the beam CB. Resolve P and Q vertically and horizontally, and we have

$$\tfrac{1}{2}P\sqrt{2}-\tfrac{1}{2}Q=0,$$
$$\text{and } \tfrac{1}{2}P\sqrt{2}+\tfrac{1}{2}Q\sqrt{3}=200;$$
$$\therefore Q=\frac{400}{\sqrt{3}+1}$$

Now resolve Q vertically and horizontally, and we get the pressure and thrust at C.

$$\text{Thrust at } C = \frac{400}{\sqrt{3}+1} \times \tfrac{1}{2} = \frac{200}{\sqrt{3}+1} = 100(\sqrt{3}-1) = 73.2.$$

$$\text{Pressure at } C = \frac{400}{\sqrt{3}+1} \times \tfrac{1}{2}\sqrt{3} + 50 = 176.79 \text{ lbs.}$$

Similarly we find thrust at $P = 73.2$.
" " pressure " $= 123.2$.

6. Statics, pages 49 and 54.

Let t be the tension of the string. Resolve t vertically and horizontally, and we get $\frac{1}{2}t$ and $\frac{1}{2}t\sqrt{3}$ respectively. Produce the reactions of the vertical wall and plane till they meet in D. Let $AC = a$. and taking moments about the point D, we have,

$$a. \tfrac{1}{2}t\sqrt{3} = \tfrac{1}{2}a. W + a. \tfrac{1}{2}t;$$

$$\therefore t = W(\sqrt{3}-1)$$

7. Statics, pages 70 and 74.

If r be the radius of the inscribed circle, then 2 area of triangle$=5ar+4ar+3ar=4a.3a$;

$$\therefore r = a.$$

Area of inscribed circle$=\pi a^2$.

Let x=distance of $C.$ $G.$ of remainder from side $3a.$; then taking moments about side $3a$, we have,

$$\pi a^2 - a + (6a + a^2 - \pi a^2)x = 6a^2 \frac{3a}{3};$$

$$\therefore x = a.$$

And if y=distance of $C.$ $G.$ from side $4a.$, taking moments about side $4a$, we have,

$$y = \frac{(8-\pi)a}{6-\pi}$$

$\sqrt{x^2 + y^2} = 1.972a$=distance of C G from right angle

8. If C be the middle point of the lever ACB; P, Q, the weights suspended at E. F;

$AC=BC=a$, then $P.EC=Q.CF=Q\ (a-CE)$, since $FB=CE$;

$$\therefore\ EC=\frac{Qa}{P+Q},\ CF=\frac{Pa}{P+Q}$$

If, when the weights are reversed, the fulcum must be placed at a distance x from C; we have (W being the weight of AB),

$$Wx+P(x+CF)=Q(CE-x);$$

$$\therefore\ x=\frac{Q.CE-P.CF}{W+P+Q}=\frac{Q-P}{W+P+Q}a.$$

No. 2.

The references are to Hamblin Smith's Hydrostatics.

1. Hydrostatics, Arts. 18—21, 24.

The pressure at any point of a fluid is measured by the pressure which would be produced upon a unit of surface, if the whole of that unit were pressed uniformly, with a pressure equal to that which it is proposed to measure. 143 lbs.

2. The whole or resultant pressure is equal to the weight of a column of the liquid having the plane area for base, and the vertical depth of the centre of gravity of the plane area for height.

6×10 in.=area of plane area; depth of centre gravity=5 inches; hence, $6\times10\times5\times\frac{1000}{1728}$=whole pressure.

3. Hydrostatics, Art. 61.

Let x=weight of cylinder, W=weight of equal bulk of water

then, $x+1\text{lb}=\frac{1}{2}W$;

and, $x=\frac{1}{3}W$;

$$\therefore\ \frac{x}{x+1}=\frac{3}{2}$$

Or thus: Since cylinder rises one-sixth of its axis when one pound is removed, therefore, 6 lbs.=weight of volume of water equal in bulk to volume of cylinder; and when weight is removed, it has a third of axis immersed, therefote, weight $=\frac{1}{3}$ of 6 lbs.

Hydrostatics, Arts. 41 and 46.

5. Since spheres are proportional to the cubes of their radii, the volume of the whole ball is eight times that of the cork, or we have 7 volumes of gutta percha to 1 of cork. Call the weight of one volume of water unity, then 1 volume of cork weighs .24 ;

$\therefore$ 7 volumes of gutta percha weigh $7 \times .98 = 6.86$;

$\therefore$ the 8 volumes weigh $6.86 + .24 = 7.10$,

and 8 volumes of water weigh 8 ;

therefore, the specific gravity of the whole is $\frac{7.1}{8} = \frac{71}{80}$, and

therefore $\frac{9}{80}$ of the ball floats above the surface.

6. Hydrostatics, Art. 73.

By the height of a column of mercury is meant the perpendicular distance between the level of the mercury in the tube and the level in the cistern. If this is measured in each case, it is of no importance whether the tube is vertical, but if the height be read from a scale attached to the tube, it is evident that this scale will only give correct results when the tube is vertical.

7. Let w=wt. of cork ; then since $\frac{\text{wt}}{\text{sp.gr}}$=vol., we have

$$\frac{150}{1.12} + \frac{w}{.24} = \frac{150 + w}{1}$$

$$\therefore\ w = 75 \times \tfrac{3}{7} \times \tfrac{3}{19}\text{lbs.}$$

If V=volume, we have

$$V \times .24 \times \tfrac{1000}{16} = 75 \times \tfrac{3}{7} \times \tfrac{3}{19}\text{lbs.}$$

8. Let x=wt. of ship and cargo.

The ship rises 1 inch in discharging 12000 lbs

$\therefore$ " 2 " " 24000 "

If upward pressure of a column of sea water$=x$

then " " " river water$=x-24000$lbs.

hence, $1.026(x-24000)=x$;

$\therefore\ x = 947076.923$ lbs.

No. 3.

1. Statics Art. 10.

2. Statics, Arts. 29 and 32.

Let the force of 50lbs. intersect the direction of P in C, then the direction of the pressure on the point B must pass through the point C (Statics Art. 34. From B draw B D at right angles to B A, meeting the direction of P in D. The triangle B D C has its sides parallel to the direction of the three forces which keep the rod A B at rest, and are therefore proportional to them. B C D is an equilateral triangle, and since B D is parallel and proportional to the 50lbs., therefore each of the other two forces is also 50lbs.

3. For the general working of such problems, see Statics, page 32.

In this case 1 and 4, 2 and 5 are opposite; the problem then reduces to three forces, each equal to 3, and making angles of 60° with each other. The counter-balancing force is 6.

4. Statics, Arts. 58 and 54.

5. Take moments about the edge of the table.

6. Statics, Art. 108.

7. Take, for example, the case in which there are 4 pulleys. Take moments about the point where the string passing over the first pulley is fastened to the rod and let $x=$ distance required from this point then, by the principle of moments, we have

$$P(2^4-1)\,x=0\,P+2.2P+4.2^2P+6.2^3P.$$

Or $$(2^4-1)\,x=2.2+4.2^2+6.2^3,$$

From which x can easily be found.

If there are n pulleys, we have

$$\left(2^n-1\right)x=2.2+4.2^2+6.2^3+\ldots\ldots\left(2^n-2\right)2^{n-1}$$

The right-hand member is a series of quantities in geometrical progression, having their co-efficients in Arithmetical

progression, the method of summing which may be learned from any of the larger Algebras.

Statics, articles 123 and 130.

In practice friction, rigidity of cords, etc., must be taken into account.

8. The gate is kept at rest by three forces; the horizontal strain on the upper hinge, the weight, and the thrust on the lower hinge passing through the intersections of these two. The forces are proportional to the three sides of the triangle thus formed, that is to 4, 4, and $4\sqrt{2}$, and since $56=4\times 14$, the strain on the lower hinge is $4\times 14\sqrt{2}=56\sqrt{2}$.

No. 4.

2. $$\text{Density}=\frac{12\times 1000+16\times 2000}{\frac{2}{3}(1000+2000)}=22$$

3. Depth of C. gravity of first square below surface $=8\frac{1}{2}$ in.
" " second " " $=x+2\frac{1}{2}$.

If w be the weight of unit of water, we have

$$3\times 8\tfrac{1}{2}\times 81w=25(x+2\tfrac{1}{2})w; \quad \therefore\ x=82.62;$$

$\therefore$ depth of side below surface $=82.62-2.5=80.12$ in.

4. Let edge of cubical vessel $=2a$, and let s_1, s_2 be the densities of water and mercury respectively.

Then pressure on upper half of a side $=2a.a.\frac{a}{2}s_1=a^3s_1$.

To find the pressure on the lower half we must first find the thickness (z) of a lamina of mercury, the weight of which is equal to the weight of water; then

$$zs_2=as_1;$$

$$\therefore\ z=a\frac{s_1}{s_2}.$$

Pressure on lower half $=2a.a\left(a\frac{s_1}{s_2}+\frac{a}{2}\right)s_2$;

$\therefore$ " " a side $=a^3(s_1+2s_1+s_2)$
" " base $=4a^2(as_1+as_2)$.

Hence, pressure on sides : pressure on base $::\ s_2+3s_1 : s_2+s_1$.

Or thus :—

S. G. of water$=1$, and of mercury$=13\frac{1}{2}$,

and w=weight of unit of water ; then we have,

Pressure on upper half of side$=2a.a.\frac{a}{2}w=a^3w$.

" lower " due to water$=2a^2.a.w.=2a^3w$

" " " " mercury$=13\frac{1}{2}.2a.\frac{3a}{2}.w$.

$$\frac{\text{Pressure on sides}}{\text{Pressure on base}}=\frac{4(a^3+2a^3+13\frac{1}{2}a^3)w}{4(a^3+13\frac{1}{2}a^3)w}=\frac{33}{29}.$$

5. By the weight of the volume of air displaced.

The S. Gr. of the diamond is less than that of the weights, and since apparent wt. of diamonds$=$wt. of air disp'd$+$real wt. of diamonds$=$constant wt.,

Then, the heavier the air the less will be the apparent wt. of the diamonds and, therefore, the better for the buyer.

6. The principle of Archimedes is "*that a body immersed in a liquid loses a part of its weight equal to the weight of the displaced liquid.*"

Let x= weight of gold in crown ;

then $w-x$= weight of silver ;

$$\text{hence } \frac{4}{77}x+\frac{2}{11}(w-x)=\frac{w}{14};$$

$$\therefore x=\frac{11}{20};$$

$\therefore$ gold : silver $::$ 11 : 9.

7. 13.39 lbs.

8. Let x be the amount of this pressure ; the air in the upper part of the tube will have a pressure represented by $\frac{15}{25}x$ and this, together with the height of the mercurial column 302, will be the pressure exerted in the interior of the tube on the level of the mercury in the bath, which is equal to the atmospheric pressure, that is $\frac{15}{25}x+302=x$, from which $x=755mm$.

9. Let d be the density of the air at first,

d_5 " " after 5 turns

d_3 " " " 3 "

Then $d_5=(\frac{10}{11})^5d$; $d_3=(\frac{10}{12})^3d$.

P, the quantity of air exhausted from pump with barrel of 1 foot capacity

=quantity at first—quantity after 5 turns.

$=10d-10d_5$.

Similarly, Q, the quantity exhausted from the other pump

$=10d-10d_3$

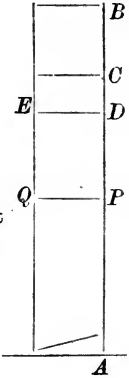
Hence,

$$\frac{P}{Q}=\frac{10d-10d_5}{10d-10d_3}=\frac{1-(\frac{10}{11})^5}{1-(\frac{10}{12})^3}=\frac{9}{10}\text{ nearly.}$$

10. Let $AB=20$ feet

$AC=17$ feet

$AD=16$ feet



(1) When the range of the piston is BD

Let P be the greatest height, and let $AP=x$,

$$\frac{\text{pressure of air in } DQ}{\text{pressure of air in } BQ}=\frac{BP}{DP}=\frac{20-x}{16-x}.$$

But pressure in $BQ+x=33$, if we assume that the water barometer stands at 33 feet,

$$\therefore \text{ pressure } DQ=\frac{(20-x)\,(33-x)}{16-x}=33\,;$$

$$\therefore\ x=10+\sqrt{-32}.$$

Here there is no greatest height, and the water flows out.

(2) When the range of piston is BC,

$$\frac{(20-x)\,(33-x)}{17-x}=33\,;$$

$$\therefore\ x=11 \text{ or } 9.$$

Hence $AP=11$ feet or 9 feet according as the piston is at B or C respectively, the valve being open.

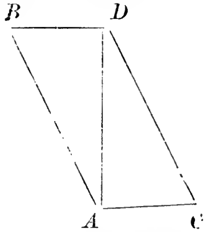
No. 5.

THE REFERENCES ARE TO KIRKLAND'S STATICS.

1. Art. 10.

If 5lbs. is represented by 1ft. 3in.,
1lb " " 3in.,
8lbs. " " 24in., or 2ft.

2. Art. 23, and Appendix, page 168.



Let $AB=2a$; $AC=a\sqrt{3}$, and $AD=a$; then the sides of the triangle ACD will represent the forces. Since $(4a)^2=(a\sqrt{3})^2+a^2$, the angle CAD is a right angle, and its sides are in the ratio of 2, $\sqrt{3}$, 1; therefore the angle ACD is 30°, and consequently $CAB=150°$.

3. This may be understood from the analogous case of the action of the wind in propelling a vessel; see example 3, page 33.

4. See example 4, page 84.

Area of triangle=36=area of square. Take moments about vertex of triangle; and if x=distance of C. G. from the vertex, we have—

$$(36+36)x=8\times 36+15\times 36;$$
$$\therefore \quad x=11\tfrac{1}{4}.$$

5. Arts. 86, 88.
6. Arts. 105, 106.
7. Let P=the power, and W=the weight.
When plane is smooth, $P=\frac{1}{2}\sqrt{2}W$.
When plane is rough, $P=\sqrt{2}W$.

$$P+\mu R=\frac{1}{\sqrt{2}}\cdot\sqrt{2}W=W;$$

but $\qquad R=W$, and $P=\frac{1}{2}\sqrt{2}W$;

$$\therefore \mu=\tfrac{1}{2}(2-\sqrt{2})=\cdot 2929.$$

8. Let AB be the lever, and C the position of the fulcrum; let $CA=x$, then $CB=10-x$. Take moments about C, and we have—

$$\tfrac{1}{2}\sqrt{3}xP=\tfrac{1}{2}(10-x)Q;$$
$$\therefore x=\frac{10Q}{P\sqrt{3}+Q}$$

9. In the figure, example 4, page 63, from B draw BN perpendicular to AO.

In this case, $R^1=\frac{W}{2}$,

Take moments about A, and we have—

$$\frac{W}{2}\cdot AG=W.AK;$$

$$\therefore AG=2AK=AN.$$

Since angle AGB=angle ANB=right angle, and $AG=AN$, then angle BAG=angle BAN; and BC is parallel to AG;

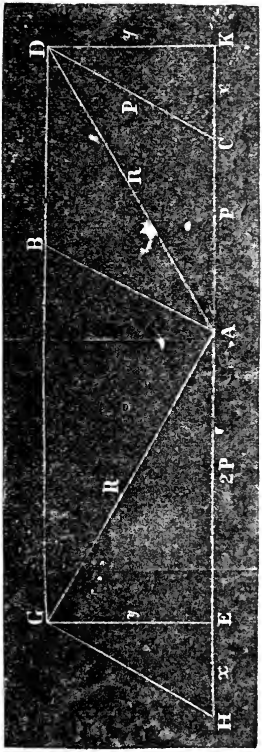
$$\therefore GAN+BAN=60^\circ;$$

$$\therefore BAN=30^\circ.$$

No. 6.

THE REFERENCES ARE TO KIRKLAND'S STATICS.

1. Arts. 15, 23, and Appendix page 164, (1).



Let $AB = AC =$ the forces, and $R =$ their resultant

Then, $R^2 = (P+x)^2 + y^2 \ldots\ldots (1)$.

Produce P backwards and make it $= 2P$, and complete the parallelogram $ABGH$

The triangle $GHE =$ triangle DCK;

therefore $R^2 = (2P - x)^2 + y^2 \ldots . (2)$

Subtracting (1) from (2), we have—

$$3P(P-2x)=0;$$

$$\therefore x = \frac{P}{2};$$

$$\therefore y = \frac{\sqrt{3}}{2}P$$

$\therefore$ the sides of triangle DCK are in the ratio of $P, \frac{1}{2}P$ $\frac{\sqrt{3}}{2}P$, or as 2, 1, $\sqrt{3}$, therefore the angle $DCK = BAC = 60°$.

3. Art. 42.

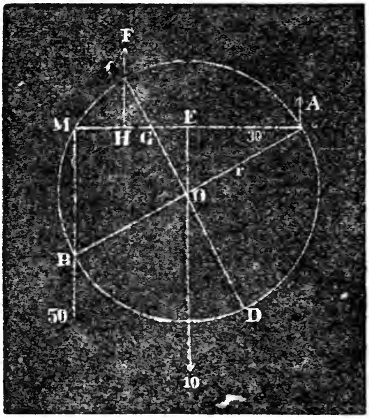
Resolving vertically and horizontally, we have—

Sum of vertical forces $= 20 + 70\sqrt{3} = a$,

" horizontal " $= 20\sqrt{3} - 10 = b$;

" resultant " $= \sqrt{(a^2+b^2)} = 143{\cdot}378$ lbs.

4. When three forces keep a body at rest each one is equal and opposite to the resultant of the other two ; then see Art. 56.



$$AB=2r, \therefore AE=\frac{r}{2}\sqrt{3} \text{ ; since angle } GOE=GCH=30^\circ$$

$$HG=\tfrac{1}{2}GC,$$

$$\text{and } GE=\tfrac{1}{2}GO:$$

$$\therefore HG+GE=\tfrac{1}{2}(GC+GO)=\tfrac{1}{2}r;$$

$$\therefore AH=\frac{r}{2}(\sqrt{3}+1).$$

Taking moments about A, we have—

$$50r\sqrt{3}+10\frac{r}{2}\sqrt{3}=F\frac{r}{2}(\sqrt{3}+1);$$

$$\therefore F=69\cdot73725.$$

5. Let W=wt which placed on the end will be on the point of overturning the beam.

Taking moments about the prop adjacent to W.

$$4W=140;$$

$$\therefore W=35.$$

6. Let AFB be the rod, of which F is the fulcrum, and let the 8 oz. be placed at A, and the 7 oz. at B.

Since $7\times 6>5\times 8$, $C.G.$ lies between A and F.

Let x=distance of $C.G.$ from A.

Taking moments about F

$$42=40+W(5-x);$$
$$2=\quad W(5-x)\ (1).$$

Interchange the weights and let F_1 be the new fulcrum.

Taking moments about F_1 we have

$$8\times 5\tfrac{6}{17}=7\times 5\tfrac{11}{17}+W(5\tfrac{11}{17}-x);$$
$$\therefore 56\ =96W-17Wx\qquad(2).$$

From these equations $W=2$ oz.

$x=4$ inches.

7. Neither gains nor loses.

8. The weight $=22$lbs,

Let $\bar{x}$=perpendicular distance of *wt.* from AB.

and $\bar{y}$= " " " " AC.

Taking moments about AB.

$$22\bar{x}=\frac{9\sqrt{231}}{4};$$

$$\therefore\quad \bar{x}=\frac{9\sqrt{231}}{88}=1\cdot554.$$

Taking moments about AC,

$$22\bar{y}=\frac{5\sqrt{231}}{10},$$

$$\therefore \bar{y}=\frac{\sqrt{231}}{44}=\cdot345.$$

9 Art. 119.

$$\frac{W}{10}=\frac{2\times 60\times 3\cdot 1614}{\frac{1}{5}},$$

$$\therefore\ W=18849\cdot 6\text{lbs}$$

10. Let a = length of beam.

x = distance of $C.G.$ from the lower end of the beam.

W = wt. of beam.

R = reaction of the floor,

and F = friction of the beam on the floor

$$=\frac{R}{2}.$$

Reaction of the wall $=\frac{R}{2}$,

Friction along " $=\frac{R}{4}$;

$$\therefore\ W=R+\frac{R}{4}=\frac{5R}{4}.\ \ \ldots\ldots(1)$$

Taking moments about the foot of the wall,

$$\frac{Ra\sqrt{3}}{2}=(a-x)\frac{\sqrt{3}}{2}W+\frac{Ra}{4}\ldots\ldots(2),$$

From these equations $x=\dfrac{a(3+2\sqrt{3})}{15}$, or the segments by the centre of gravity are as 43:57 nearly.

No. 7.

THE REFERENCES ARE TO HAMBLIN SMITH'S HYDROSTATICS.

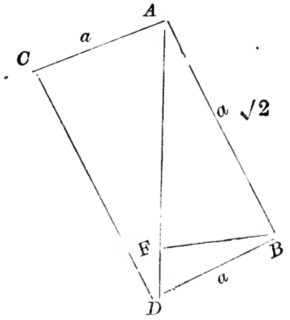
1. Art. 21.

The pressure at any point of a fluid is measured by the pressure which would be produced upon a unit of surface, if the whole of that unit were pressed uniformly with a pressure equal to that which it is proposed to measure.

2. Arts. 23, 92. 16lbs.

3. The pressure of a fluid on any surface is equal to the weight of a column of the fluid, the base of which is equal to the area of the surface, and the altitude equal to the

depth of the centre of gravity of the surface below the surface of the fluid.



Let $AC=a$ = length of side of cube,

then $AB=a\sqrt{2}$ = length diagonal of side,

and $AD=a\sqrt{3}$ = length diagonal of cube.

From B draw BF perpendicular to AD.

AF may be found by similar triangles, or as follows:

$$AB^2-BD^2=AF^2-FD^2=(AF+FD)(AF-ED)$$
$$=a\sqrt{3}(AF-FD);$$

$$\therefore \quad AF-FD=\frac{a^2}{a\sqrt{3}}=\frac{a}{\sqrt{3}},$$

$$AF+FD=a\sqrt{3}=\frac{3a}{\sqrt{3}};$$

$$\therefore \quad AF=\frac{2a}{\sqrt{3}}, \text{ and } \therefore \ FD=\frac{a}{\sqrt{3}}$$

Distance of C. G. of upper side, AB, from $A=\frac{1}{2}AF=\frac{a}{\sqrt{3}}$.

By symmetry, distance of C. G. of lower side from $D=\frac{a}{\sqrt{3}}$;

$\therefore$ distance of C. G. of lower side from $A=a\sqrt{3}-\frac{a}{\sqrt{3}}=\frac{2a}{\sqrt{3}}$.

But A is a below surface of water ; therefore,

distance of C. G. of upper side below surface $=a+\frac{a}{\sqrt{3}}$,

" " lower " " $=a+\frac{2a}{\sqrt{3}}$,

or, $\frac{\text{upper side}}{\text{lower side}}=\frac{\sqrt{3}+1}{\sqrt{3}+2}=\frac{\sqrt{3}-1}{1}$

4. Art 65.

Wt. of iron in water $=6.7$lbs.

Wt. of iron + wt. of wood $=5.3$lbs. in water ;

$\therefore$ Pressure of wood upwards $= 1.4$lbs. $=$ difference between wt. of wood and wt. of equal bulk of water ;

$\therefore$ wt. of equal bulk of water$=7$lbs $+ 1.4$lbs.$=8.4$lbs.

Hence S. G. $=\frac{7}{8.4}=\frac{5}{6}$.

5 S. G. of chain$=\frac{2000}{153}$.

v=vol. of brass ; and v_1=vol. of gold ;

then $7.8v+19.3v_1=\frac{2000}{153}(v+v_1)$;

from which we find vol. brass:vol. gold::9529:8066.

6. Art 67.

Wt. of metal=14.85 grams.

When the metal is placed in the lower cup 2.03 grams must be added to make the instrument float at the same level as before,

therefore wt. of water displaced$=2.03$ grams ;

hence S. G. of metal$=\frac{14.85}{2.03}=7.31$ "

7. Art. 80.

In the first case a litre of air$=\frac{755}{760}\times 1.293$ grams.

second " " $=\frac{770}{760}\times 1.293$ "

$$\text{change in weight} = \frac{770-755}{760} \times 1.293 = .025 \text{ grams.}$$

8. Art. 71.

Wt. of cork=wt. of water displaced+wt of air displaced; hence as the air displaced by the cork diminishes, the water displaced must increase; that is, the cork will sink in the water.

9. Art. 86.

Air, like other fluids, transmits pressure equally in all directions; therefore, in this case, a pressure of 77lbs. is transmitted to every 3.5 sq. in. of the receiver:

pressure on every sq. in $= 77 \div 3.5 = 22$lbs.
total pressure in air chamber$=(22+15)$lbs.$=37$lbs.

10. 15 lbs. pressure = column of water $\frac{144 \times 16 \times 15}{1000} =$ 34.56ft. high.

and since the volume varies inversely as the pressure, we have—

$34.56 : (600+34.56) :: 2 : x$;
$\therefore x = 36\frac{13}{18}$ cubic inches.

No. 8.

NOTE—The references are to Kirkland's Statics.

1. The force of gravity diminishes slowly from the pole to the equator. A mass of matter which would compress a spring with a force equal to that of 194lbs. at the equator, would act upon it with a force of 195lbs. at the poles. This difference would not, of course, be perceived by the ordinary moαe of weighing by the balance, as both weights of the body would be similarly and equally affected.

2. Articles 2, 8, 9, 10, and note, page 20.

3. Article 23, and Appendix I.

4. The resultant of any two of the forces is equal and opposite to the other force.

Construct a parallelogram whose sides are 7 and 14 and the included angle 120°. Bisect the side 14, and complete the parallelogram. The resultant of 7 and 7 acting at 120° is 7; combine this resultant with the remaining 7 of the 14, and the required result is attained.

5. On each post the resultant pressure is 200 lbs., acting towards the centre of the circle.

6
$$P-Q=28,$$
$$P^2+Q^2=52^2$$
$$P^2-2PQ+Q^2=28^2$$
$$\therefore \quad 2PQ=52^2-28^2=80\times 24\ ;$$
$$\therefore \quad P^2+2PQ+Q^2=4624\ ;$$
$\therefore$ $P+Q=68$, from this and the first equation, P and Q may be easily obtained.

7. Article 29. The three lines represent the direction of the forces, but not their lines of action.

8. Let P, Q, R be the forces; then we have $P^2+Q^2=R^2$, and $\frac{R}{Q}=\frac{5}{3}$, from which the required ratio is found.

9. Produce the weight backwards till it meets P in the point C. From B draw BD at right angles to AB, then the three sides of the triangle BCD will represent in magnitude and direction the three forces which keep the rod BA at rest. AD is bisected in C, and since the side opposite the 30° is half the hypothenuse, $BD=CD$, therefore, the triangle BCD is equilateral, and hence, BC, the reaction of the hinge, $=DC,=P,=50$ lbs.

10. See Ex. 5, page 26.

No. 9.

(The references are Kirkland's Statics.)

1. Art. 29.

2. When the weight is vertically over the centre of the street, complete the parallelogram of which the adjacent sides are each 35. One of the diagonals of this parallelogram is the width of the street 50 feet, and the other, which cuts it at right angles, is found to be $20\sqrt{6}$.

Now, if the weight $=20\sqrt{6}$, tension $=35$;

$$\text{"} \quad = \quad 1, \quad \text{"} \quad =\frac{35}{20\sqrt{6}}$$

$$\text{And} \quad \text{"} \quad = \quad 8, \quad \text{"} \quad =\frac{8\times 35}{20\sqrt{6}}$$

$$=\frac{14}{\sqrt{6}}.$$

3. Art. 38. Let AB be the given force. From A draw AC, making an angle of 60° with AB; and from B let fall a perpendicular, BD, on AC. Complete the rectangle of which AD, BD are adjacent sides. The adjacent sides of this rectangle terminating in A are the required forces.

4. The outer cords evidently make angles of 45° with the horizon, and 90° with each other. Resolve each of the inner cords along each of the outer cords, and we get $100+50+50\sqrt{3}$. This force makes an angle of 45° with the vertical and horizontal lines. Resolve vertically and horizontally. The horizontal forces neutralize each other. The vertical components are:

$$2\left(\frac{150+50\sqrt{3}}{\sqrt{2}}\right)=(150+50\sqrt{3})\sqrt{2}=334.$$

There is 18 cwt. $=1800$ lbs., acting downwards; therefore, the strain $=1800-334=1466$ lbs.

5. The statements in this problem will appear evident enough by drawing the figure and carefully examining it.

6. Art. 63.

7. 12 cwt.

8. 364 lbs.

No. 10.

(The references are to Kirkland's Statics.)

1. Arts. 2, 18, 20.

2. Suppose the wt. to be drawn out 4 feet. We have then a right-angled triangle, the hypothenuse of which is 20 feet, the base 4 feet, and the perpendicular, $8\sqrt{6}$. If F be the force required, then by the triangle of forces,

$$\frac{F}{4}=\frac{300\text{ lbs}}{8\sqrt{6}}.$$

3. The fourth force will be represented in magnitude and direction by CA.

4. Ex. 2, page 32.

Resolving vertically and horizontally, we find the magnitude of the resultant $= 52{\cdot}44$; the direction of the resultant cannot be found without a knowledge of trigonometry.

5. Resolving vertically and horizontally, we find the algebraic sum of the vertical components$=5\frac{1}{2}-7=-\frac{3}{2}$; algebraic sum of the horizontal components$=2\frac{1}{2}\sqrt{3}-3\sqrt{3}=-\frac{\sqrt{3}}{2}$; hence the resultant will be between the forces 6 and 7; and its magnitude$=\sqrt{3}$.

Since the horizontal component$=\frac{1}{2}\sqrt{3}$, and the hypothenuse $=\sqrt{3}$, the adjacent angle is therefore $= 60$, (Art. 28); the force 6 makes an angle of 30°, with this same horizontal line the resultant will therefore make with the force 6 an angle of 90°.

6. The wt. will divide the rope into two parts of 6 feet each. If the parallelogram of which these two parts are adjacent sides be completed, the diagonal will evidently be 6. Hence if the wt. were 6 cwt, each of its components would be 6 cwt; but the wt. is 1 cwt., each of its components is 1 cwt., that is, the tension is 1 cwt.

7. Arts. 51, 52, 53, 56.

8. Take moments about the point of the lever which the finger supports, and wehave

$$P \times 2 = 16 \times 6 \text{ oz.}$$
$$\therefore \; P = 48 \text{ oz.}$$

The finger has to support the 6 oz., and the 48 oz., or 54oz.

9. 9 feet from the end near the heavier boy; 6 feet from the rail.

10, Art. 50, 45, 46.

11. Distance from the side of square = $\cdot 7384a$.

Chemistry No. 1.

1. Roscoe, pages 140, 142.

Atomic Weight is the smallest proportion by weight in which an element enters into or is expelled from a chemical compound,—the smallest weight of hydrogen so entering or leaving a chemical compound being taken as unity.

2. *Chemical Compounds* possess the following characteristics:

(1) Possess properties which differ entirely from their constituents.

(2) No purely mechanical means will suffice to separate the constituents of a chemical compound from each other.

(3) But the most important characteristic is that the constituents are bound together in a fixed and definite proportion by weight.

Air a mechanical mixture:—

(1) In mixing oxygen and nitrogen in their proper aerial proportions, neither change of volume, nor heat, nor electricity results.

(2) The relative amounts of oxygen and nitrogen present in the atmosphere are not their combining weights, nor any multiples thereof.

(3) When air is shaken up with fresh boiled water, the gases dissolve in the proportion of 1 of nitrogen to 1.87 of oxygen, and not in the proportion in which they are contained in air.

3. The term "atomicity" is employed to indicate the greatest number of atoms of one kind or another with which a given atom is ever observed to unite.

4, 5, 6, and 7 may be answered from any good text-book, such as Roscoe's or Miller's Inorganic Chemistry.

8. The *crith* is the weight of one litre or cubic decimetre of hydrogen at 0° C., and a pressure of 760 millimetres of mercury. The following is Dr. Hofmann's description of the of this unit:—

"The actual weight of this cube of hydrogen, at the standard temperature and pressure mentioned, is .0896 gramme; a figure which I earnestly beg you to inscribe, as with a sharp graving tool, upon your memory. There is probably no figure in chemical science more important than this one to be borne in mind, and to be kept ever in readiness for use in calculation at a moment's notice. · · For example, the relative volume-weight of chlorine being 35.5; that of oxygen, 16; that of nitrogen, 14; the actual weight of 1 litre of each of these elementary gases, at 0°C. and 760 mms. pressure, may be called respectively, 35.5 *criths*, 16 *criths*, and 14 *criths*."

9. Roscoe, page 254.

By dividing each percentage by the atomic weight, we find the simplest formula to be NH_2O; but no such body is known; $NH_2O = N_2H_4O_2 = NH_4NO_2$ = ammonium nitrite.

No. 2.

1. An *Acid* is a compound, containing one or more atoms of hydrogen, capable of displacement by a metal presented to it in the form of a hydrate.

Examples.—$HCl + KOH = KCl + H_2O$.
$HNO_3 + NaOH = NaNO_3 + H_2O$.
$H_2SO_4 + Ca(OH)_2 = CaSO_4 + 2H_2O$.

A *base* is a compound body capable of being converted into a salt by the action of an acid, and of thereby more or less neutralizing the reactions of the acid.

A *salt* is a compound formed by the mutual action of an acid and a base.

2. The steam is decomposed; the oxygen unites with the red-hot iron, forming magnetic or black oxide of iron (Fe_3O_4); cupric or black oxide of copper is formed (CuO).

3. (1) Roscoe, p. 18. (2) Add zinc to the caustic potash and boil; we have $2KOH + Zn = ZnK_2O_2$ (potassic zincic oxide) $+ H_2$.

4. Towards the end of the last century, Lavoisier poured some mercury into a flask, with a long narrow neck, which he placed on a sand-bath, so that its temperature might be constantly maintained at about 660° F., for several weeks. The mercury combined with the oxygen of the air, being converted into a red powder (HgO). By heating this oxide in a tube of hard glass to a temperature approaching a red heat (1000°F.) it is decomposed into mercury and oxygen.

$$2HgO = 2Hg + O_2.$$

5. This subject will be discussed in next month's EXAMINER.

6. 7. Roscoe.

$$\frac{2\ NaNo_3}{170} + H_2SO_4 = Na_2SO_4 + \frac{2HNO_3}{126};$$

$\therefore$ 1 ton yields $\frac{126}{170}$ tons nitric acid.

9. $2HCl + Zn = ZnCl_2 + H_2$.

1 litre of hydrogen weighs .0896 gramme.

1 cubic centimetre " $\frac{.0896}{1000}$ "

250 " " $\frac{250 \times .0896}{1000}$ gramme.

Now in evolving 1 gramme of hydrogen, $\frac{65}{2}$ grammes of zinc are dissolved;

$\therefore$ in evolving $\frac{250 \times .0896}{1000}$ " $\frac{65}{2} \times \frac{250 \times .0896}{1000}$

grammes of zinc are dissolved.

10. *Absolute atomicity* is the maximum equivalence of an element: *latent atomicity* is that portion of a polyad element's combining power, which in certain of its combinations is unexercised; *active atomicity* is the exercised portion of an element's atom fixing capability, and this, with its latent atomicity, makes up its absolute power.

Carbon is a tretrad element, its *absolute* atomicity is four, and we see the whole of its combining power exercised in

marsh gas (CH_4) and in carbon dioxide (CO_2) whilst in carbon monoxide (CO), it plays the part of a diad element, having two bonds *active* and two *latent*, combined with and satisfying each other.

No. 3.

1. See EXAMINER No. 2.

The molecule of water (H_2O) is 18 grams.

Hence 18 grams of water yield 2 grams of hydrogen.

"	1	"	.1111	"
and	1	"	.8889	" oxygen.

1000 cubic centimetres of hydrogen at 0°C. and 760 mm., weigh .08936 grams. Then what volume would .1111 grams of hydrogen occupy?

The result is 1243.28 cc. of hydrogen
and 621.713 cc. of oxygen.

2. (2). $2\ MnO_2 + 2H_2SO_4 = 2MnSO_4 + 2H_2O + 2O_2$

It is usual to say that the Manganese Dioxide acts by *Catalysis* which refers to the phenomena which takes place when effects are brought about by the mere presence of a substance which itself undergoes no perceptible change. In this case, the $KClO_3$ is decomposed at a much lower temperature than would be required if MnO_2 were present, and yet the MnO_2 is found, after the experiment, in the same state as at the commencement. It may, however, undergo a temporary alteration. We know that this MnO_2 is capable of taking up more oxygen and of forming manganic acid (H_2MnO_4), and it is possible that when heated with $KClO_3$, the MnO_2 may absorb oxygen from the $KClO_3$, and pass into the state of the higher oxide which is immediately decomposed, the oxygen being evolved and the MnO_2 returning to its original state. This same effect is noticed if $KClO_3$ be mixed with CuO, Fe_2O_3, &c., all of which are known to be susceptible of higher oxidation. The oxides of zinc, magnesium, &c., on the contrary, which do not form higher oxides,

do not facilitate the decomposition of the Potassium Chlorate

8. Cold sulphuric acid has no action on copper.

When heated, the action of the copper on the sulphuric acid, is probably first of all

$$Cu+H_2SO_4=CuSO_4+H_2.$$

But the hydrogen never escapes, for at the moment of its liberation, it acts upon another portion of the sulphuric acid. and reduces it :—

$$H_2+H_2SO_4=2H_2O+SO_2.$$

So that the gas actually evolved is sulphur dioxide.

Nitric Acid is what is called an *oxidizing agent*, that is, it readily gives up part of its oxygen to any oxidizable element, and is thus broken up. Zinc with very dilute nitric acid gives the following reactions :—

$$4Zn+10HNO_3=4Zn(NO_3)_2+NH_4NO_3+3H_2O.$$

With moderately dilute cold nitric acid we have :—

$$4Zn+10HNO_3=4Zn(NO_3)_2+N_2O+5H_2O.$$

With somewhat less dilute nitric acid, we get :—

$$3Zn+8HNO_3=3Zn(NO_3)_2+2NO+5H_2O$$

Concentrated nitric acid dissolves zinc but slightly, the nitrate being very sparingly soluble in nitric acid.

No. 5.

1. *Hydrogen.*—Water dissolves a trace of hydrogen ; 100 volumes take up 1.93 volumes of the gas, its solubility being unaffected by the temperature of the solvent.

Carbon dioxide combines with water, forming carbonic acid.

$$CO_2+H_2O=H_2CO_3.$$

Ammonia combines with water, forming ammonium hydrate.

$$NH_3+H_2O=NH_4OH.$$

Calcium carbonate.—Water dissolves a very small quantity of calcium carbonate, about 2 grains in a gallon.

Nitrogen trioxide combines with water, forming nitrous acid.

$$N_2O_3+H_2O=2HNO_2.$$

Sodium and *Potassium* each replace one atom of the hydrogen in the water, forming hydrates and setting the hydrogen free.

$$Na_2+2H_2O=2NaOH+H_2.$$

2. *Carbon* burned in Oxygen gives Carbon dioxide; $C+O_2=CO_2$.

Sulphur burned in Oxygen gives Sulphur dioxide; $S+O_2=SO_2$.

Phosphorus burned in Oxygen gives Phosphorus pentoxide; $P_2+O_5=P_2O_5$.

Sodium burned in Oxygen gives Sodium oxide; $Na_2+O=Na_2O$.

3. (1.) Make a mixture of iron filings with about two-thirds of their weight of sulphur. A greenish-gray powder results, but distinct patches of iron and sulphur can be easily recognized in it, not only with the aid of a magnifying glass, but also by stirring some of the powder into a considerable quantity of water, when the heavy particles of iron fall quickly to the bottom of the vessel, while the lighter sulphur more slowly subsides, and collects as a distinct layer. Now heat the substance *very strongly* in a tube of hard glass; the mixture becomes pasty, and then glows for a short time. Cool and remove the resulting substance from the tube. When examined with a magnifying glass no particles of iron or sulphur can be detected. The iron and sulphur are no longer separable by mechanical means. The iron and sulphur have chemically combined and formed a new substance, iron sulphite.

(2.) Gunpowder affords an exceedingly good instance of the difference between the effects of a mechanical mixture and a chemical combination. The *nitre* may be washed out with water, the *sulphur* may be washed out with carbon disulphide, leaving the *charcoal* behind undissolved. If, however, the mixture is fired, the three solids disappear, and are suddenly converted into an enormous volume of gaseous matter, the new substances produced possessing properties totally distinct from those of the *nitre*, *sulphur* or *charcoal*.

4. Roscoe, page 92.

7. Roscoe, page 60. It is much better, however, to represent the reaction in two stages.

(1) $3Cu+6HNO_3=3Cu\ (NO_3)_2+3H_2$.

(2) $3H_2+2HNO_3=2NO+4H_2O$.

8. Ammonium carbonate, $(NH_4)_2CO_3$, and Nitric acid, HNO_3, give Ammonium nitrate, NH_4NO_3, and Carbonic acid.

$$(NH_4)_2CO_3+2HNO_3=2NH_4NO_3+H_2CO_3=$$
$$2NH_4NO_3+H_2O+CO_2.$$

It resembles oxygen :

(1) In rekindling a glowing chip of wood when plunged into it.

(2) In supporting combustion with nearly equal energy

It differs from oxygen :

(1) By being much more soluble in water.

(2) In extinguishing a feeble flame of sulphur.

(3) It does not form red fumes with nitrogen dioxide (NO).

(4) It is not absorbed by potassium pyrogallate.

(5) When phosphorus is burned in it, the residual gas is of the same volume as the original gas.

9. Pour a little water into a bottle ; put some sulphur in the deflagrating spoon and burn it in the same bottle ; dip a glass rod into nitric acid and hold it in the bottle ; repeat this several times, and shaking up each time. Now add a little barium chloride, a white precipitate will appear, showing the presence of sulphuric acid.

10. Add hydrochloric acid, and filter.

11. The different modes of chemical action may be classified as follows :

(1) When elements or compounds combine directly with each other.

$$NH_4+HCl=NH_4Cl,$$
$$S+O_2=SO_2.$$

(2) When compounds are split up into their elements, or into less complex components.

$$HgO=Hg+O.$$
$$NH_4NO_3=N_2O+2H_2O.$$

(3.) When one element or group of elements displaces another element or group of elements.

$$2HCl+K_2=2KCl+H_2.$$
$$CuSO_4+Fe=FeSO_4+Cu.$$

(4) When elements or groups of elements in one body are exchanged for other elements or groups of elements in another body.

$$CuCl_2+H_2S=CuS+2HCl.$$
$$KNO_3+H_2SO_4=KHSO_4+HNO_3.$$

(5) When the elements of a compound are re-arranged, as in the conversion of starch into sugar. There is no good example of this in inorganic chemistry.

No. 6.

The answers to nearly all the questions on Chemistry will be found in the two preceding numbers and in the present number.

8. Pictet, of Geneva, effected the liquefaction of oxygen under a pressure of 300 atmospheres, and at the temperature produced by the evaporation of liquid carbon dioxide in a vacuum. On removing the pressure a jet of liquid oxygen escaped from the tube. By a double circulation of sulphur dioxide and carbon dioxide, the liquefaction of the carbon dioxide was effected at a temperature of $-85°$ F., and at a pressure of 4 to 6 atmospheres. This liquid carbon dioxide was then passed along a tube about 12 feet long, communicating with the air-pump. By forming a vacuum with the pumps the carbon dioxide solidifies. Through the interior of this tube a second and smaller tube passes, through which a current of oxygen, set free in a strong vessel, can be passed.

No. 7.

1. See EXAMINER for October.

2. See EXAMINER, page 264.

3. Hydrogen is usually prepared by decomposing sulphuric acid by zinc. The usual impurities are arseniuretted hydrogen, $\mathbf{AsH_3}$, when the zinc or acid contains arsenic; phosphoretted hydrogen, when either contains phosphorus; nitrous fumes, when the acid contains nitric acid or nitrates; sulphur dioxide and sulphuretted hydrogen, when these gases are contained in the acid or when hot, even diluted, sulphuric acid is allowed to come in contact with the metal.

4. When zinc is heated with a strong solution of caustic potash or soda, the zinc replaces the hydrogen, forming a compound of zinc oxide and potash, $\mathbf{K_2ZnO_2}$. This process yields an inodorous gas:

$$2KOH + Zn = K_2ZnO_2 + H_2.$$

5. 39·04 parts by weight of potassium, 22·99 parts of sodium, and 7·01 parts of lithium (these numbers being the accurate atomic weights) will yield the same volume of hydrogen. Hence, if equal weights be taken, lithium will yield the most, sodium will come next, and potassium will yield the least.

6. See EXAMINER for November, p. 291.

7. See EXAMINER, page 293. When the sulphur is burnt in air, nitrogen will be in the jar as well as sulphur dioxide.

8. Cavendish first ascertained that by the combustion of two volumes of hydrogen and one volume of oxygen pure water and nothing else is produced. He did not, however, fully understand the results, and in 1783, Lavoisier gave the true explanation of the composition of water.

For methods of obtaining hydrogen from water, see EXAMINER, present number.

Oxygen may be obtained from water—

(1) By electrolysis.

(2) By passing steam and chlorine through a red-hot porcelain tube:

$$2H_2O + 2Cl_2 = 4HCl + O_2.$$

9. The impurities of water are either (1) inorganic; (2) organic. Suspended matter may be removed by filtration. See EXAMINER for May, page 158.

10. In 1772, Rutherford discovered nitrogen, and in 1774, Priestly discovered oxygen. In 1774, Lavoisier proved that air consists of a mixture of these gases, in the proportion of 1 volume of oxygen and 4 volumes of nitrogen. The remainder of this question will be found in Roscoe or Miller.

11. Air is a *mechanical mixture* of nitrogen, oxygen, and other gases, as proved by the following circumstances:

(1) On mixing oxygen and nitrogen in their proper aerial proportions, neither change of volume, nor heat, nor electricity results.

(2) The relative amounts of oxygen and nitrogen present in the air are not their combining weights, nor any multiple thereof.

(3) When air is shaken up with fresh boiled water, the gases dissolve in their normal proportions, *i. e.*, as 1 of hydrogen to 1·87 of oxygen; and not in the proportion in which they are contained in air, viz., as 4 of nitrogen to one of oxygen.

12. The impurities contained in commercial nitric acid are the lower oxides of nitrogen; chlorine and iodine (derived from the alkaline chlorides and iodides in the nitre); sulphuric acid, iron, alumina, potash, and soda salts.

No. 8.

1. A candle consists essentially of carbon and hydrogen; there is a little oxygen.

Of the two elements of the gas, hydrogen has the greatest affinity for oxygen, and therefore burns first, momentarily setting free the carbon, which is sprinkled in a fine powder through the burning gas. This is at once intensely heated, and each glowing particle becomes the centre of radiation, throwing out its luminous pulsations in every direction. The sparks last, however, but an instant, for the next moment the charcoal combines with the oxygen. Other particles succeed, which become united in turn, and hence, although the sparks are evanescent, the light is continuous.

2. Roscoe, page 49.

Gay-Lussac, in 1808, enunciated the following law, which served as a powerful argument in favour of Dalton's Atomic Theory: This law states that *The weights of the combining volumes of the gaseous elements bear a simple ratio to their atomic weights.*

32 grams of sulphur combine with 56 grams of iron filings; hence 100 grams of iron filings will combine with 57·14

grams of sulphur. There is, therefore, 42·86 grams of sulphur in excess.

3. Mercuric Oxide **HgO**, Manganese Dioxide, $\mathbf{MnO_2}$, Red Lead $\mathbf{Pb_3O_4}$, Barium Dioxide, $\mathbf{BaO_2}$. For equations see EXAMINER for November.

4. All the metals which sulphuretted hydrogen will precipitate from neutral or alkaline solutions, that is, metals of the "Iron Group," will give off hydrogen upon the addition of sulphuric acid.

Zinc, Iron, Manganese, Cobalt, Nickel, Chromium belong to this group.

For equations see last month's EXAMINER.

5. Animals inhale oxygen and exhale carbon dioxide. Plants absorb carbon dioxide and give off oxygen, the volume of oxygen given off being equal to the volume of carbon dioxide decomposed. The following equation probably represents the reaction :—

$$12CO_2 + 11H_2O = \underset{\text{Sugar.}}{C_{12}H_{22}O_{11}} + 12O_2.$$

6. See last question. Also by the principle of gaseous diffusion.

7. See last month's EXAMINER.

8. See Roscoe, page 41.

9. See Roscoe, page 63.

10. See Roscoe, page 67.

11. Deducting 3 per cent. from 150 grams, we have 145½ grams. From every 100 parts of pure calcium carbonate we obtain 44 parts by weight of carbon dioxide, and therefore from 145·5 grams of carbonate we get 64·02 parts of carbon dioxide.

But a litre of carbon dioxide weighs 22 criths or 1·9712 grams, and in 64·02 grams there are 64·02÷1·9713=32·4

litres at 0°C, and 760 mm. pressure, which at 745 mm. and 15°C. will be increased to 34·8 litres:

$$\frac{32 \cdot 4 \times 760 \times 288}{745 \times 273} = 34 \cdot 8$$

which is therefore the volume obtained.

English Literature.---No. 1.

1. See notes to Canto I., Taylor's ed., where the construction of the Spenserian stanza is fully explained, and some of the writers named.

Iambic Tetrameter.

2. See analysis of each canto at the beginning of the notes on that canto, Taylor's ed.

6. See notes, Stanza 16, Canto III.

7. An *objective* writer pictures and describes *outward life* as perceived by the sense or realized by the imagination. He dwells almost exclusively on scenes apparent to an ordinary observer, and does not trouble himself with the great world of thought *within.* The *subjective* writer on the other hand, instead of the outward sense, describes the various feelings and thoughts which it occasions in the mind.

8. Taking the lines in order, the first is an example of *Oxymoron,* or the figure by which is meant the saying of that which appears foolish, yet is to the point ; other examples are, "Cruel kindness," "A pious fraud," etc. The second is an example of *Metonymy,* the effect being put for the cause ; the third of *Alliteration* the fourth of *Pleonasm.*

No. 2.

2. In the Spenserian stanza Scott refers metaphorically to the prominent qualities set forth by the incidents in the canto of which it forms the introduction ; it also serves to connect the sentiment of the preceding canto with that of the following one.

3. For a use he makes of middle rhyme see the ballad of Alice Brand, Canto IV. St. XII : Now must I *teach* to hew the *beech.*

6. (*a*) Chief, meaning Roderick—Metonymy
Torrent, daughter—Personification.
Sounding shore—Metaphor.
Mighty lakes—Hvperpole.
Silver—Metaphor.
Ceaseless—Enallage.
Lines....Rome—Allusion.
Empress—Antonomasia.
Eagle-wings—Metonymy.
Lowland-warrior—Metonymy and Euphemism.
Bold Saxon—Exclamation.
Inversion and alliteration occur frequently.

(*b*) & (*c*) See notes to Taylor's edition.

7. See notes to St. 4. Canto IV.

No. 3.

NOTE—THE REFERENCES ARE TO TAYLOR'S LADY OF THE LAKE.

2. (*a*) *Caitiff* signifies a wretch, one of a base abject disposition.

(*b*) Sleep.

(*c*) *Court of Guard*, the Guard-room.

(*d*) *Harness*, armor.

(*e*) *Clouded*, swarthy.

(*f*) See notes to St. 3, Canto VI.

(*g*) "The jongleurs, or jugglers, used to call in the aid of various assistants to render these performances as captivating as possible. The glee-maiden was a necessary attendant. Her duty was tumbling and dancing."

(*h*) *Damosel* is from the French *damoiselle*, the diminutive of dame. *Errant* is used in its literal sense, wandering.

See notes St. 9. Canto VI

(*i*) *Guerdon*, reward, a gift.

(*j*) *Antique garniture*, old furniture, See notes St. 12, Canto VI.

3 (*a*) So, &c—Simile.
His, referring to sun—Personification.
Summer tears—Metaphor.
Soldiery—Abstract for concrete—Metonymy.
As Simile &c.—
The ship—General for particular—Synecdoche.
Her, referring to ship—Personification.
Pine,—Roderick's crest for his clan—Metonymy.
Bough—Metaphor.

6. The *Review*, begun in 1704 by Defoe, was the first English serial. It was followed in 1709 by the *Tatler* which appeared twice a week and was written by Richard Steele and Joseph Addison, Steele contributing by far the greater number of papers.
The *Spectator* next appeared in 1711. This was a daily publication edited by Addison and Steele. In 1713 the *Guardian*, a daily sheet, made its appearance. It was also edited by Addison and Steele. To Samuel Johnson we are indebted for the *Rambler* and the *Idler*. In Scotland, Henry Mackenzie and a number of others conducted the *Mirror* (1759) and the *Lounger* (1765).

No. 4.

1. Note the following differences between prose and poetry:—
   (*a*) They differ in *form*.
      1. As to the words employed.
      2. As to the arrangement of words.
      3. Poetry permits the largest possible use of figures of speech.
      4. Poetry has generally a certain mode of presentation peculiar to itself into which the words are thrown. This is called versification.

   (*b*) They differ as to the *sentiment*.

In poetry this is always more elevated, more impassioned, and more imaginative.—*De Mille's Rhetoric.*

8. Bred to no business, &c.—*Hysteron Proteron.*

The order of thought is reversed, and that is put first which should come last.

English Grammar.--No. 1.

3. Such general terms as *motion*, *animal*, *color*, and *crime*, are of classic origin ; the special terms, *creep*, *step*, *slide*, *stagger*, etc., *man*, *boy*, *cow*, *horse*, etc., *blue*, *red*, *white*, *green*, etc,, *murder*, *steal*, *theft*, *rob*, etc., are of A. S. origin.

5. See Mason, art. 176, etc.

6. *Brethren* and *kine* are examples of a double plural, formed by a change of vowel and the suffix *en*. *Children* has also two marks of the plural, viz., the suffix *er* and *en*.

7. See Mason, art. 373, for the parsing of man, and also for the kind of phrase *man to man* is.

8. (*d*) by supplying the ellipsis, the meaning becomes apparent. John loves James better than he (John) loves him (another party). John loves James better than he (another party) loves James.

(*e*) We say the lion's mane, when we are speaking about different parts of the same animal ; the mane of the lion, when we are speaking of the manes of different animals.

9. *That* is generally used instead of *who* or *which*

(1) After adjectives in the superlative.

(2) When the relative refers to two antecedents, one requiring who and the other which : as, the man and the horse that we saw driving on the ice were drowned.

3. Generally after *same*.

10. (*d*) The antecedent of they, is clearly each, which is used in the singular number only.

No. 2.

2. See Art. 83.

4. See Art. 402.

5. (*a*) See Art. 202.

(*b*) Loving is a verbal noun.
(*c*) Loving is an adjective.
(*d*) Respecting is a participle
(*e*) Maintaining is a verbal nouu.
(*f*) Speaking is a participle used absolutely. See Art. 372.

7. (*c*) *Pageant* from Lat. pangere to fasten, fix: From pangere, the Low Lat. term, *pagina*, for stage or scaffold is derived. Subsequently the term applied to the stage was used to designate the exhibition on the stage, and lastly any fantastic exhibition, cf. *Person* from *persona*, a mask, then the person masked, and lastly an individual.

(*d*) Bent—Catachresis.
Sad—Transfer of epithet
All is true, &c.—Ellipsis.
Fiery—Metaphor.
Steel—Metonymy.
I, only, I—Epizeuxis.
Bride of Heaven—Metaphor and Euphemism.
How excellent:—but, &c.—Aposiopesis.

(*d*) Should evidently read, "This word I have found, only in Spencer."

In (*b*) the first verb should be in the past perfect tense as the first event had been finished before the other took place, hence "I had written, &c."

(*c*) The ellipsis of the second verb is incorrect as the author has changed the number of the nominative from the plural to the singular. It should read: "Many sentences are miserably mangled and the force of the emphasis *is* totally lost.

(*d*) As the construction cannot speak to make a sound, it is the *quality of the sound, not the manner of sounding* which ought to be expressed; hence this sentence should be "This construction sounds rather harsh."

(e) It, representing *whatever a man conceives clearly*, is redundant, as it is improper to use both a noun and its pronoun as the object of the same verb.

No. 3.

1. In English the use of case-endings has to a great extent been replaced by the use of prepositions (Art. 65.) and by the position of the noun in the sentence.

The absence of case-endings requires us to place the words in a sentence according to a fixed method, in order to avoid obscurity of meaning. Thus, when the case-ending indicated the case, in the sentence John loved James, it was at once clear which one of the names was in the subjective relation and which one in the objective relation to the verb. The order of words was of no importance as far as determining the case was concerned. The sentence might have been written James John loved, &c., the inflection in each case indicating which word was in the nominative case and which in the objective. But now if the order is changed the meaning becomes doubtful.

2. Arts. 130, 132, 138, 148, 152.

3. Arts. 413 and 151.

4. Art. 222. The origin of the term *weak* is from the fact that certain verbs formed their past tense by using an *auxiliary* which is still retained under the form *d*, *ed*, *t* &c.

5. Art. 358.

6. Art. 355.

7. (*a*) *Is* to be avoided, because the predication must be understood to be confined to extravagance; *as well as* merely refers us to an illustration.

(*b*) As the idea conveyed by complete is "that of a state of fulness having no deficiency," it should not be compared. Good usage however, permits of such comparison.

Art. 113.

(c) As there are two subjects the verb should be plural.

(d) If the antecedent of who be I, the sentence is correct. If man be regarded as its antecedent, it should read who *is* a Jew.

(e) As the subjects are singular and connected by nor the verb should be singular.

(f) Me is grammatically incorrect. Some writers regard me as another form of the nominative and thus defend this use of the word.

Who should be whom because the objective case is required after prepositions.

(g) This is correct. The infinitive is used after certain verbs, as "to hear," without its sign "to."

8 *To-morrow.* In Early English "*to* was often used with a noun to form an adverb." (Abbott's *Shakespearian Grammar*, Art., 190.)

Early to bed. "To is a preposition."

Go to now. "To" is used adverbially as in "heave to," "come to"

Such a to-do—*To* and *do* together for a noun.

To is the sign of the infinitive—the two words have become one compound word. See Earle's Philology, Art. 455.

9. See the preliminary notice to Mason's Grammar, p. 1, 2, 3 and 4.

10. *Live* is in the Subjunctive mood. Art. 195.
There is an adverb of place.
Themselves is in the Objective case after deemed.
Part is objective complement to deemed. Art. 391.

(c) Pageant—Metonymy.
Long live....James—Exclamation.
Commons' King—Synecdoche and Antonomasia.
Peer and Knight—Enallage—Singular for the plural.
You—Enallage—Definite for indefinite pronoun.

The mean..disdained—Hypallage.
Tower—Synecdoche.

(*d*) Quaint, Lat. *cognitus*, known.
Hostage, *ob* and *sedeo*. The 'h' is prosthetic. It has no connection with *hostis*, an enemy.

No. 4.

THE REFERENCES ARE TO MASON'S GRAMMAR.

1. Arts. 52 and 53.

For *alms, summons, eaves, riches* see art. 60; for *dice*, 55. *Banns, costs* and *weeds* are plural.

2. Art. 288.

Connectives include all words which serve to join sentences whether principal or subordinate, and thus include *Relative pronouns* and *Relative adverbs* as well as conjunctions.

3. Arts. 114, 115,

4. Arts. 33, 200, 183, 247, 227,

5. A *Phrase* is two or more words correctly put together but not making a statement. Thus, A man of wisdom will succeed; *of wisdom* is a phrase. This is made at the place where we carry on our business; *at the place where we carry on our business* is a phrase.

A *clause* is a subordinate sentence.

Art. 343.

6. See *Preliminary Notice*, page 3

7. (*a*) *Such* should be followed by *as* in the clause with which it is connected except when the clause expresses a consequence, when *that* is employed. Hence *which* should be replaced by *as*.

(*b*) Since *either* is singular (art, 175) its predicate should be singular.
Hence *are* is incorrect, it should be *is*.

(*c*) Supply *the* before propagating, or omit *of* before vice. Art. 470.

(*d*) *Instantly* is a repetition of *as soon as* and, hence, should be omitted.

(*e*) This sentence contains the "*and which*" error. This error is found when the words "*and which*" are employed " in a sentence not containing, in the preceding part of it, the word "*which*," either expressed or understood," referring to the same antecedent. The sentence is also faulty in having no antecedent for *therefore*. It would be correct thus :—

"The rules, definitions, and observations which are more important, and which are therefore the most proper to be committed to memory, are printed in larger type.

8. (*b*) A *hybrid* is an English root with a foreign prefix or suffix, or a foreign root with an English prefix or suffix. e.g. *disown*.

9. (*a*) *Would*, is Past Indicative of "will."

(*b*) *Me* is a Reflective Pronoun. Art. 177.

(*c*) *Wot* is Present Indicative. Its past tense is *wist*. See art. 245.

11. The prefix "*be*" is used for the following purposes:—

1. Prefixed to instransitive verbs, it makes them transitive, as bemoan, bewail, &c.
2. Prefixed to some transitive verbs, it changes the object of the transitive relation, as behold, beseech, behave.
3. Prefixed to some transitive verbs it gives the idea of more intensity and completeness, as bepraise, besmear.
4. Prefixed to nouns and adjectives, it forms verbs, as bedew, beguile, bedim, becalm, behead, befriend, &c.
5. It is used in certain combinations to form adverbs and prepositions, as beneath, below, beside. &c. Angus' Hand-book, Art. 148.

No. 5.

[NOTE.—THE REFERENCES ARE TO MASON'S GRAMMAR.

1. Compare the expressions: "The king's picture" and "The picture of the king"; "The Lord's Day" and "The Day of the Lord."

2. For the *three* relations expressed by the objective case, see arts. 79 and 80.

3. Arts. 188 and 195. It is so called from usually occurring in subordinate sentences.

6. (*a*) James works *better than he* does.
   (*b*) John loves James *better than him.*
   (*c*) John, *than whom* a truer friend never lived, died.
   (*d*) The house, which you purchased, *and which* I thought so cheap, caught fire.
   (*e*) James is *as good as I* am at algebra.
   (*g*) Robert considers him *as good as me.*
   (*h*) John is the older, but James is *the wiser man.*
   (*i*) *Would that* wars would cease.
   (*j*) He drove *sixty-head* of cattle to market yesterday.

8. (*a*) *Other* should be followed by *than.*
   The sentence should be "with *no aid but* the notes," or "with *no other aid than* the notes."
   (*b*) *No* should be *not.*
   (*c*) It is not correct to say "we had read." It should be "we *would* sooner read," &c.
   (*d*) Our most careful writers, when speaking of inanimate objects, use "*of which*" instead of "*whose.*"

Read: "How can we define that *of which* we cannot comprehend the being, the action," &c.?

(*f*) *Fewer* should be used when speaking of *numbers*, *less* when speaking in *bulk*.

(*g*) Read "*of overcoming it*," to make the construction correspond to the former part of the sentence.

9. Construe as follows, and the parsing becomes evident: "All of Douglas I have left *were reft with* that gallant pastime."

That is a subordinate conjunction, &c.

Day is subject nominative to cut.

But is equivalent to *who not*, and by some grammarians is called a negative relative.

Grim and *still* are adjectives in the predicative relation to spirit.

These words which properly describe Roderick's body are by a figure (transfer of epithet) applied to his spirit.

10. *Aghast*—the *h* is excrescent, probably to harmonize its spelling with ghost.

Aghast is from A. S. *gæstan* to terrify.

Could—See Art. 242, note.

Island—"The *s* is ignorantly inserted owing to confusion with isle, a word of French origin." Island is from A. S. *ea*, water, and *land*.

12. *Untie* and *undo*. *Un* reverses the meaning of the root word.

Unkind; *un* means not.

Unloose; *un* has no meaning.

No. 6.

11. (*b*) The comparative degree should be followed by *than*. Hence, "no sooner blown *than* blasted."

(*c*) "Let thee and *me*."

(*d*) "When he we serve is away."

(*e*) —"He I accuse has entered," &c.

(*f*) The sentence as it stands makes Moses meeker than himself; "All the men" must be so changed as not to include Moses.

(*g*) Let I make a covenant is certainly not English.

(*h*) "I shall attempt neither to palliate nor deny."

(*i*) This sentence is faulty in that the pronoun, "they" is used in a slovenly manner. Any rendering which will make it clear that the antecedent of "they" is "population" and not "priests" will improve it.

No. 7.

1. (*b*) *Hy-me-ne'-al*, *mu-se'-um*, *re-cess'*, *ar-tif'-i-cer*, *al-ly'*.

5. (*c*) (1) *Auburn* is in the Nominative of Address, being the name of the thing addressed.

(2) *Plato* is also in the Nominative of Address.

(3) *John* is the Possessive case, John is the name of the owner of the coat.

(4) *Chairman* is in the Objective case, being what Mason calls the Objective complement. See Art. 395

(5) *Man* is in the Nominative case, being the Subjective complement. See Art. 393.

(6) *This* is in the Nominative case, the phrase 'this said' being in the *Absolute Construction.*

10. (*a*) This is correct, grammarians are generally agreed that the subject of the absolute phrase is in the Nominative case in English.

(*b*) As the clause is meant to be restrictive, the relative "*that*" would be more correct than "*which.*"

(*c*) Dr. Craik remarks on this, "Of course, it should be *than I.* But the personal pronouns must be held to be, in some measure, emancipated from the dominion or tyranny of Syntax. Who would rectify even Shelley's bold,

'——lest there be
No solace left for *thou* and *me!*'

The grammatical law has so slight a hold that a mere point of euphony is deemed sufficient to justify the neglect of it."—*English of Shakespeare*

(*d*) This is correct. In cases where the two names are almost synonymous, the verb is often made singular.

(*e*) This may be justified on the ground that there is *an ellipsis*, and that *and* joins two statements. "There was racing and *there was* chasing."

(*f*) Bain says that this is incorrect. He says it would be correct to say "two pounds and two pounds are four pounds, but with numbers in the abstract, what is meant is that the numerical combination of two and two is the same as four.

(*g*) This is incorrect. The meaning is that "two taken twice makes four." Two being the name of a number is used as a singular noun.

No. 8.

2. (*b*) See Mason, Art. 413 for the two uses of the relative. When the relative is used as a mere connective or as Mason has it with a *continuative* force it connects two co-ordinate sentences— *e. g.* I met the watchman, *who* told me there had been a fire. At school I studied geometry, *which* I found useful. The relative may even introduce an adverbial clause, *e. g.* It is no use asking John, *who knows nothing about it.*

10. (*a*) Whom should be who. (See Mason, Art. 393.)

(*b*) This is correct. Whom is the object of the verb "love," The sentence in full being "They whom the gods love die young."

(c) This is correct. Him is used reflexively. "Of a pillar" would be better than "pillar's" but the latter may be allowed for the sake of the metre, &c. (See Mason, Art. 78.

Geography,--No. 1.

1. It is 12 o'clock with B ; A looks south ; B, north.

2. The Gulf stream in the Atlantic and the Japanese Current in the Pacific distribute a large quantity of heat over these oceans, and the return trade winds, the prevailing direction of which is from the south-west, carry this heat eastward and thus heat up the western sides of the continents ; the eastern sides are moreover, cooled by the polar currents, which from the diurnal rotation of the earth tend westward and thus keep as close to the eastern coasts as possible.

3. The anti-trade winds, which blow in a direction opposite to the trade winds counterbalance the retarding influence, of the latter.

4. See Sullivan's General Geography.

5. The climate of a country is its condition in regard to temperature, moisture, and prevailing winds.

The general law is that the climate of a place depends upon its latitude.
The chief modifiers of this law are the following : (1) Altitude, (2) Prevailing winds, (3) Ocean currents, (4) Proximity of mountain ranges, &c.

6. 11 hr. 2 min, 3½ sec.

9. On the revocation of the Edict of Nantes four vessels sailed from Holland carrying about 150 French Huguenots ; these were landed in South Africa, and soon co-mingled with the Dutch already there. The Boers are the descendants of this mixed race. In 1835-6 the Boers emigrated from Cape Colony northward and founded Pietermaritzburg. They emigrated again and again until finally they took up their abode to the North of the Vaal, a tributary of the Orange River.

No. 2.

3. The earth rotates on its axis in 23 hrs. 56 min. 4 sec. This is a *sidereal day*. If the earth did not move forward in its orbit, this would also be the *solar day*, but from the time the sun is on the meridian till it is on the same meridian again, the earth has moved forward, and after completing its rotation on its axis it requires nearly 4 minutes more to bring the same meridian under the sun.